# Application of Neural Networks to The Non-Stationary Heat Conductivity Problems

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*Abstract* - **The paper is devoted to application of recurrent artificial neural networks to solution of non-stationary heat conductivity problems. A recurrent neural network has been constructed for implementation of the multi-grid method. Space-time discretization of the problem was established using the projective-grid scheme of the finite element method (FEM).**

*Keywords* - **delamination, unilateral contact, variational inequalities, finite element method, evolutionary algorithms**

## I. PROBLEM STATEMENT

The object of our investigation is the non-stationary heat conductivity problem which can be described as follows

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} + Lu(x,t) = f(x,t), & \\ Lu := -\dfrac{\partial}{\partial t}\left[ m \dfrac{\partial u}{\partial x} \right] + b\,\dfrac{\partial u}{\partial t} + su & \forall (x,t) \in (0,1)\times(0,T], \\ u(0,t) = u(1,t) = 0 & \forall t \in [0,T], \\ u(x,0) = u_0(x) & \forall x \in [0,1], \end{cases} \tag{1}$$

where $u=u(x,t)$ is the unknown temperature distribution, $\mu=\mu(x)$, $\beta=\beta(x)$, $\sigma=\sigma(x)$ and $u_0=u_0(x)$ are defined nondimensional functions (physical interpretation see in [4]).

In contrast to elliptic PDEs, in the case of the heat conduction equation we must consider changes in time as well as in space. For example, the parabolic equation (1) can be solved by substituting divided differences for the partial derivatives or using the FEM. But in this study we propose the alternative approach based on the neural networks (NNs) [1] in contrast to well-known numerical techniques. For space-time discretization of the given problem (1) we use the project-grid scheme of the FEM [3]. There are many advantages to the use of NNs to solution the initial-boundary problems [4].

## II. TEST EXAMPLE

Some possibilities of effective application of the proposed approach are demonstrated. On the concrete examples (heat flux for a plate subject to a fixed boundary temperatures and heated plate with an insulated edge) it has been shown, that the results of calculation using new approach give good approximations to exact solutions [4]. The non-stationary problem of heat conductivity transversally isotropic thin plate with the mixed boundary conditions of heat exchange was also solved. Note that, in this study the linear distribution of temperature over the plate thickness and the convective heat exchange with external medium by the Newton's law are assumed. With time, the solution eventually approaches the steady-state level.

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With the aim of illustrating the numerical accuracy of the proposed technique, as a test problem we consider initial-boundary problem for a insulated uniform thin rod with a length of $l$ positioned between two bodies of constant but different temperatures $T_1$ at $x=0$ and $T_2$ at $x=l$, respectively. That is, the boundary conditions are fixed for all times $u(0,t)=0$, $u(1,t)=1$, where $u=(T-T_1)/(T_2-T_1)$ is the dimensionless temperature. In addition, the temperature distribution as a function of position at an initial time is known: $u(x,0)=\sin px + x$.

In this case the heat-conduction equation has the second derivative in space and the first derivative in time, that is

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0$$

Note that, this initial-boundary problem can be solved analytically. Solution can be expressed as follows $u = \exp(-p^2 t)\sin px + x$.

Table 1 presents a comparison of the exact analytical solution $u$ with the numerical solution $u_h$ using of a recurrent neural network ($r = h^2/2$, $q = 1/2$, $e = 10^{-4}$, $Dt = h/2$). The numerical results obtained on the presented NN show a good approximation to exact analytical solution.

*Table 1:*

**Numerical results at** $x = 0{,}5$

| $t$ | $u_h$, $N = 401$ | $u$ | $(u-u_h)$´$10^8$ |
|---|---|---|---|
| 0,1 | 0,87270278 | 0,87270784 | 506,0 |
| 0,3 | 0,55177091 | 0,55177327 | 236,0 |
| 0,5 | 0,50719133 | 0,50719188 | 55,0 |
| 0,7 | 0,50099892 | 0,50099903 | 11,0 |
| 0,9 | 0,50013876 | 0,50013878 | 2,0 |

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