Formalization of Variation Process of Information Networks' Users' Quantity

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Abstract – **In this paper the analysis of the various functions in the view of usage for the describing of variation process of information networks' users' quantity is given.**

Keywords – **Information Network, Function, Network's Development.**

I. Introduction

From the point of view of prevalence of the offer over demand in the networks' market, the potential user has possibility to choose from numerous networks with a various set of services which are given by a selected network. For today there is a big number of information, telecommunication and other networks, at connection to which the user considers a quantity of criteria, having analyzed and having compared which, chooses to what network it is more favorably to be connected by him. For definition of regularities of users' number's change depending on concrete conditions various mathematical models are used. The analysis of models which adequately describing measurement of users' requirements on connection to the network for various conditions of this network's performance and development is made in the work. The problem's decision includes research of requirements' model of network's development at the expense of its capacity's, volume's and kinds of given services' increasing. Such research provides the analysis of the mathematical functions which use is possible for the description of variation process of users' quantity connected to the network during the different periods of its existence. It is carried out by methods of the mathematical and functional analysis.

II. Main Part

Models which describing measurement of users' requirements should uniquely define demand for network services in each instant. That is, if T – the researched period of time – discrete or continuous set, the model of requirements has to represent certain everywhere certain and unambiguous correspondence $D:T \quad \mathbf{R}^{\quad 0}$. Without limiting a generality, we can consider that T is numerical set [1]. Thus what there was a model, it can mirror the validity only with some accuracy. Therefore, for the description of real process of a network's growth we will use asymptotic denotations [2].

In overwhelming majority of the works devoted to the information network's (IN) optimization model of requirements of network's development is described by linear function from time of kind $D(t) = b + kt,\ k \quad 0$, where b – shift parameter; k – angular coefficient [3]. The shift parameter defines value of function in point $t = 0$ and allows to set some reference value of network's capacity. The angular coefficient, numerically equal to a tangent of a corner between the function's plot and abscissa axis, defines rate of function's increase or decrease, that is network's growth rate. Functions of such kind vary strictly monotonously, and with constant rate, that directly follows from a constancy of the first derivative. Constant and unlimited function's increase or decrease allows to use this function for modeling of the network's development's stable periods, in particular it can concern the initial stage of network's development when its capacity evenly grows. The linear dependence can be used for rather short period of time.

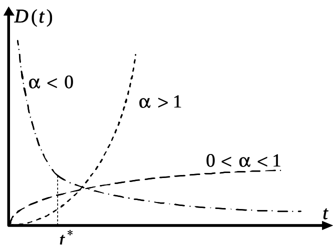

Fig.1 An exponential function

Usage of an exponential function of kind $D(t) = t^{\alpha},\ \ \alpha \quad \mathbf{R}, \alpha \quad 0, \alpha \quad 1$ allows to describe non-uniformly varying process. Generally, it is possible to take advantage of polynomial function of kind

$$D(t) = a_n t^n + a_{n-1} t^{n-1} + ... + a_1 t + a_0 = \\ = \sum_{i=0}^{n} a_i t^i,\ n \quad 0,1, a_i \quad \mathbf{R} \quad i = \overline{0,n} \quad . \qquad (1)$$

However entered asymptotic denotations do such functions by equivalent to functions of a kind (1). Really,

$$P(t) \quad R[t] O(P(t)) = t^{\deg P(t)}, \qquad (2)$$

where $R[t]$ – a ring of real polynomials from one character over the field; $\deg P(t)$ – degree of a polynomial $P(t)$.

In other words,

$$n, a_1, a_2, ..., a_n \quad \mathbf{R} \quad D(t) = a_n t^n + a_{n-1} t^{t-1} + ... + a_1 t + \\ + a_0 = \Theta(t^n) \qquad (3)$$

From the resulted expression it is visible, that members of low degrees, practically do not influence character of process, on matching with a high member, and they can be discarded.

Functions of a kind t^{α} can be used for the description of a wide range of processes, depending on parameter α. In particular at $\alpha > 1$, function beyond all bounds increases with accruing rate and can have a number of extremes and excess points. Such functions approach for the description of process of rapid development in which the number of users fast increases, and, the further, the more intensively. At $0 \quad \alpha \quad 1$ return process is observed. Function grows in the beginning quickly enough, but with increase in value of argument its growth is decelerated. Nevertheless, it is impossible to speak about limitation of such kind's functions. How value A was great, always there would be a value of argument t_A, $t_A = A^n : D(t_A) = t_A^{\frac{1}{n}} = \sqrt[n]{A^n} = A$,

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corresponding to it. Functions of such kind are useful at modeling of the network's development's durable period, with fast enough rates of increase in the beginning and step-by-step decelerated rates in development.

At negative values α, the exponential function will decrease. At $\alpha < 0$ function behaves specifically enough, and can approach only for the description of process of intensive drop of demand for services. Nevertheless, having supplied function in negative coefficient, it is possible to achieve return result $\Theta(-t^{\alpha} + c)$, where c – some positive number defining a shift parameter. Thus, it is convenient to set initial capacity of a network before its upgrade.

The updated exponential function does not decrease any more, and increases, and in the beginning rather high rates. After passing of «saddle point», growth becomes not so considerable and with increase t there will be less and less notable. Despite some similarity in behavior of considered function and function t^{α} at $\alpha \quad (0,1)$, it is necessary to mark their basic difference. Accordingly $D(t) = -t^{\alpha} + c$ models saturation at $t \quad +$, while t^{α}, $\alpha \quad (0,1)$ – only decreasing growth. Saturability of function $D(t) = -t^{\alpha} + c$ can be useful at modeling of the late or durable periods of the network's development when requirements come in due course to a constant. Thus intensity of function's growth to the saddle point underlines an acceptability of the similar kind functions for the description of the long development's period in which take place also rapid growth in the beginning, and saturation in the end of the researched period.

Possibility of usage indicative logarithmic and various type of trigonometrical functions is researched in the work too. The provided research has shown, that the most convenient for the description of a wide range of processes is logistics function.

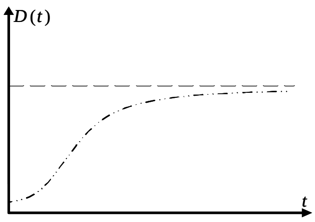

Fig. 2 A logistics function

Usage of logistics function of kind $D(t) = \Theta(\frac{1}{1 + a^{-t}})$ conveniently for the description of developing processes, in the presence of growth limitations. Logistics function possesses an excess point. To the excess point function is convex downwards, that successfully enough characterizes initial development of the network's services' market, and after the excess point function becomes convex upwards, that quite corresponds to market saturation. One more advantage of logistics function is simple enough adapting of the schedule to modeled process, by means of introduction of coefficients and constants. Coefficient a corresponds to limiting value of the value of saturation $\lim_{t \quad +} D(t) = \frac{a}{1 + b \lim_{t \quad +} e^{-ct}} = a$. Coefficient b is used for the description of entry conditions of process, whereas $D(0) = \frac{a}{1 + b}$. That is if $\tilde{D}_0$ – the initial value of requirements and $\tilde{a}$ – the saturation value, is enough to select $\tilde{b} = \frac{\tilde{D}_0}{\tilde{a}} - 1$ and then for $\tilde{D}(t) = \frac{\tilde{a}}{1 + \tilde{b}e^{-ct}}$ relations $\tilde{D}(0) = \frac{\tilde{a}}{1 + \tilde{b}} = \frac{\tilde{a}}{1 + (\frac{\tilde{D}_0}{\tilde{a}} - 1)} = \tilde{D}$ $\tilde{D}(t) \quad {}_{t \quad +} \quad \tilde{a}$ will take place. Coefficient c characterizes growth rate of function at increase t. Thus the increase in value of coefficient leads to an expedition of network's capacity's growth, and coefficient reduction, accordingly, to growth deceleration. By means of combinations of constants a, b and c it is possible to receive various variants of logistics function's behavior, that in a combination to asymptotic character of its behavior makes its rather convenient for modeling of considered processes.

In [4] it is shown that the main tags of logistics process are: positive value of the process's characteristic in an initial instant; rather fast growth of a curve in an initial stage of process; excess point presence; slow growth of a curve after the excess point; asymptotic approximation of process to a saturation limit $a = \frac{\eta}{|\varepsilon|}$. Mathematical conditions of logistics character of process are $\varepsilon = 0, \zeta = 0$, $\eta > 0$, $a > Y(0)$. Depending on concrete values of logistics function's parameters and relations between them it is possible to have various kinds of the network's development:

– absence of the network's development takes place under a condition $\eta + \zeta t + \varepsilon Y = 0$;

– unlimited growth of the network corresponds to a condition $\eta + \zeta t + \varepsilon Y > 0$;

– unlimited decrease – $\eta + \zeta t + \varepsilon Y < 0$.

In the absence of development $Y = const$, at unlimited growth $Y = Ae^{\lambda t}$, and at unlimited decrease $Y = Ae^{-\lambda t}$, where $\lambda = \eta + \zeta t + \varepsilon Y$.

III. Conclusion

The made analysis has allowed to use the various functions describing behavior of requirements model of the network's development for modeling of various situations, taking place on the concrete IN.

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