Decision Rules of Signals Recognition Comparison by Results of Statistical Modelling

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Abstract - **In the given paper the estimation of reliability of a decision, made in accordance with results of statistical simulation, on that a rule of signals recognition at a given sample size provides us with the smaller probability of an error of recognition then the other do has been carried out.**

Keywords - **Signals recognition, decision rule, statistical modelling, error of recognition, decision rules comparison.**

I. Introduction

When we deal with the problem of choosing the best decision rule among already existing, we frequently have to use subjective considerations [1]. It makes urgent the problem of prospecting for some objective criteria that will allow us to compare decision rules of signals recognition by results of their statistical modelling.

II. Derivation of Basic Expressions

We are going to consider the problem of choosing the best rule between two ones. Let us set the following events: A, decision rule R_1 has recognized a realization correctly; B, decision rule R_2 has recognized a realization correctly. The diagram in Fig. 1 shows all the possible events.

Denote by $p_1 = P(A \cap \overline{B})$ and $p_2 = P(\overline{A} \cap B)$ the probabilities of the events that consist respectively in the following: the rule R_1 has recognized a realization correctly while R_2 has failed and vice versa. Then the probability of the fact that either both of the rules R_1 and R_2 have recognized realizations correctly or both of them have failed, equals $P[(A \cap B) \cup (\overline{A} \cap \overline{B})] = 1 - p_1 - p_2$. At given p_1, p_2 the simultaneous probabilities of getting k_1 times into the area $A \cap \overline{B}$, k_2 times into the area $\overline{A} \cap B$ and $n - k_1 - k_2$ times into the area $(A \cap B) \cup (\overline{A} \cap \overline{B})$ are described by multinomial distribution Eq. 1:

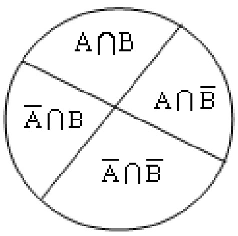

Fig. 1 The diagram of the possible events that take place during recognition of realizations by two decision rules.

$$P(k_1, k_2, n \mid p_1, p_n) = \\ = \frac{n!}{k_1! k_2! (n - k_1 - k_2)!} p_1^{k_1} p_2^{k_2} (1 - p_1 - p_2)^{n - k_1 - k_2}. \quad (1)$$

On the contrary, if the experiment showed that there were k_1 realizations from the overall number n in the area $A \cap \overline{B}$ and k_2 in the area $\overline{A} \cap B$, then the mutual probabilities p_1, p_2 of these events could be found with Bayes' formula Eq. 2:

$$P(p_1, p_2 \mid k_1, k_2, n) = \frac{P(k_1, k_2, n \mid p_1, p_2) \cdot P(p_1, p_2)}{\sum_{p_1, p_2} P(k_1, k_2, n \mid p_1, p_2) \cdot P(p_1, p_2)}, \quad (2)$$

where $P(p_1, p_2)$ is the absolute simultaneous probability of the values p_1, p_2.

Suppose that the probabilities p_1, p_2 are uniformly distributed in some area. Let $k_1 > k_2$, then the accuracy of the hypothesis on that the rule R_1 is better than the rule R_2 is given by the expression below Eq. 3:

$$P(p_1 > p_2 \mid k_1, k_2, n) = \frac{\sum_{p_1 > p_2} P(k_1, k_2, n \mid p_1, p_2)}{\sum_{p_1, p_2} P(k_1, k_2, n \mid p_1, p_2)}. \quad (3)$$

The curves, plotted on the formula Eq.3 at $n = 100$, are shown in the Fig. 2. The curve at the very bottom corresponds to $k_1 = 0$, while the curve at the very top corresponds to $k_1 = 5$. As it goes from the Fig. 2, the posterior probability of the event $p_1 > p_2$ decreases as k_2 grows.

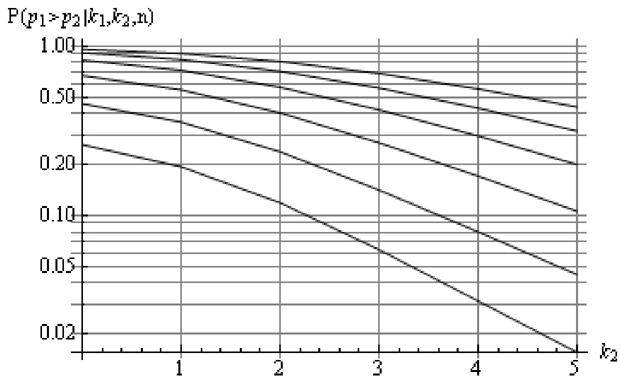

Fig. 2 Dependence $P(p_1 > p_2 \mid k_1, k_2, n)$ on k_1, k_2.

III. Conclusion

The problem of determining the accuracy of decision we take by results of statistical modelling, that some decision rule of signals recognition at a given size of sample provides us with the lesser probability of error of recognition than the others do, has been considered.

References

[1] К. Фукунага, "Введение в статистическую теорию распознавания образов", М.: Наука, 1979, P. 368.

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