Analysis Algebraic System of Arguments for Prime Size DHT

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Abstract - **Arguments of discrete harmonic transforms (DHT) for prime size specifying partial case of algebraic systems - abel groups is analyzed. Features of generate elements this algebraic systems are determined.**

Keywords - **algebraic systems, cyclic group, primitive elements, arguments, discrete harmonic transforms.**

I. INTRODUCTION

Discrete transforms and convolutions are the main operations and key tools in signal processing. The efficient computation of discrete harmonic transforms can perform by means of convolutions as fast transform technique[1]. Papers [2] use the following index mappings on base primitive root *g* of cyclic group. Further analysis of algebraic systems and specific primitive roots are important.

II. ANALYSIS OF BASIS DHT

The discrete harmonic transform is defined in matrix form:

$$X = W^* x, \qquad (1)$$

where W(kxn) – a basis square matrix: (n,k=0,(1),...,N-1), x(N) and X(N) –columns of input and output data;

$W(k,n)= \cos(2\pi kn/NT)+j \sin(2\pi kn/NT)$, DFT (discrete Fourier transform);

$W(k,n)=cas(2\pi kr/NT)=(\cos(2\pi kn/NT)+\sin(2\pi kn/NT))$, DHT (discrete Hartley transform);

$W(k,n)=c(n)x(n)\cos[\pi(2k+1)n/2NT]$, DCT (discrete cosine transform), where

$$c(n) = \begin{cases} 2^{-1/2}, & \text{if } n=0 ; \\ 1, & \text{otherwise,} \end{cases}$$

N– size of transform; T- time of quantization.

To perform analysis arguments $\varphi=2\pi(n\times k)/N$, (n,k=1,(1),...,N-1) basis of DHT. Arguments can be present the multiplications of integer number $(n\times k)$ without allowing $\Delta\varphi=(2\pi/N)$. Discrete harmonic functions are periodic relatively N, then elements of basis matrix of arguments degree $(N\times N)$ are equal

$$\mathbf{a_{kn} = (n \times k) \bmod N,} \qquad (2)$$

where a_{kn} integer values of arguments of basis matrix for n=1(1)N-1 row and k =1(1)N-1 column.

The n, k $\in$(1,2,..., N-1) and elements the matrix of arguments a_{kn} belong the set (1,2,..., N-1) accordingly (2). Therefore matrix of arguments degree $(N\times N)$ conforms the table algebraic operation (*= (n x k) mod N) of multiplication by modulo N.

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The algebraic system <N-1,*> with operation on set (1,2…N-1) corresponds equivalent basis matrix of discrete harmonic transform. In case size of transform N is prime algebraic system <N-1,*> is Abel group. Besides algebraic system <N-1,*> with prime N presents cyclic group and table of operation is Hankel circular matrix [3]. Elements of cyclic group are equal natural powers of generate element $\alpha \in G$. Generate element of cyclic group is primitive and α is not singular. Primitive element will be α^{N-1} also. Therefore all elements of cyclic group can be determined the powers of primitive element. Non-primitive elements our cyclic group to generate part of set and other part of set forms by multiplication on two elements of generating set by modulo N.

Analyze of Hankel matrix of arguments degree $(N\times N)$ as substitution π_i for each row (column) a_i, $i\in\{1,2,\ldots,x\}$ to first row (columns) of matrix [3], where N is prime. Summation of substitutions $\{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5,\ldots\pi_x\}$ form cyclic group. Quantity and numbering generate and non-primitive elements match for substitutions and operation (2).

III. CONCLUSION

In results of analysis the values of arguments the basis matrix DHT for prime size:

- set (1,2,..., N-1) of element of arguments and multiplicative operation is Abel group;
- quantity primitive elements of Abel group determine according theorem by Lagrange and can be equal the function by Euler;
- generation of cyclic group using non-primitive elements and other part of elements Abel group forms by multiplication on two elements by modulo N;
- quantity and numbering generate and non-primitive elements is equivalent for group <N-1,*> and group of substitution π.

Results of analysis use for forming matrix with circular correlations or cyclic convolutions on base primitive and non-primitive elements from the basis matrix DHT of prime N size.

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