Applying the Difference Operators for Surfaces Approximation with Given Accuracy in Nodes

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Abstract – **A method of surfaces approximation with given accuracy in nodes by difference operator using the methods of interval data analysis described in this paper.**

Keywords - **Interval data analysis, Surface approximation, Difference operator.**

I. Introduction

One from the problems in computer graphics is minimizing the time for three-dimensional images forming. This problem directly associated with forming the surfaces which are components of graphic objects. Mainly for solving such problems the approximation approaches are used. They are based on the criteria of mean square deviation or maximum deviation between the table-given function and the approximating function which represent the surface. Actual is applying other criteria for efficient using the resources of graphics pipelines. That criteria make possible to present a surface by approximation function with different accuracy in the nodes.

The large number of methods used in computer graphics for surfaces approximation are exist. But the approximation's quality with the given different accuracy in the nodal points is actual task.

One from the approaches for solving this task is the method of interval data analysis with applying two-dimensional polynomial splines. Research of these methods which implemented to software showed their main limitations: approximation's accuracy is so low on the bounds of approximation region, applying the multidimentional splines requires a multidimensional segmentation and complex conditions of connections on the bounds of segmentation. One from the possible ways for this tasks solving is applying the difference operators for approximation with given accuracy in nodes using the methods of interval data analysis.

II. Statement the Task of Approximation by the Difference Operator

Let's some surface is defined by table-given function:

$$z_{i,j},\ i=0,...,I-1,\ j=0,...,J-1. \qquad (1)$$

Let's approximate this table-given function by linear difference operator in the general kind:

$$\tilde{z}_{i+1,j+1} = \vec{g}^T \cdot \vec{f}(z_{0,0},...,z_{i,0},...,z_{0,j},...,z_{i,j}),\ i=0,...,I-1,\ j=0,...,J-1 \qquad (2)$$

where $\vec{f}(z_{0,0},...,z_{0,i},...,z_{j,0},...,z_{j,i})$ is a fixed vector of basis functions that defines the structure of difference operator; $z_{i+1,j+1}$ is a function value in the nodes with discrete coordinates $(i+1;j+1)$; $\vec{g}$ is unknown parameter vector of difference operator.

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Then this surface $z_{j,i}$ will approximate with a given accuracy in nodes which is given by an error $\Delta_{i,j}$. These errors are different for each node:

$$\left|z_{i,j} - \tilde{z}_{i+1,j+1}(\vec{x}_i)\right| \le \Delta_{i+1,j+1},\ i=0,...,I-1,\ j=0,...,J-1, \qquad (3)$$

Under these conditions will obtain inclusion in interval kind

$$\tilde{z}_{i+1,j+1} \in [z^-_{i+1,j+1}; z^+_{i+1,j+1}],\ i=0,...,I-1,\ j=0,...,J-1, \qquad (4)$$

Thus the data of table-given function are in interval kind.

Substituting in Eq. (4) instead of $\tilde{z}_{i+1,j+1}$ the Eq. (2) will get the interval system of nonlinear equations

$$\begin{cases} z^-_{0,0} \le z_{0,0} \le z^+_{0,0} \\ z^-_{1,0} \le \vec{g}\cdot\vec{f}_{1,0}(z_{0,0}) \le z^+_{1,0} \\ z^-_{2,0} \le \vec{g}\cdot\vec{f}_{2,0}(\vec{g}\cdot\vec{f}_{1,0}(z_{0,0})) \le z^+_{2,0} \\ \vdots \\ z^-_{i+1,0} \le \vec{g}\cdot\vec{f}_{i+1,0}(\vec{g}\cdot\vec{f}_{i,0}(z_{0,0},...,z_{i-1,0})) \le z^+_{i+1,0} \\ \vdots \\ z^-_{i+1,1} \le \vec{g}\cdot\vec{f}_{i+1,1}(\vec{g}\cdot\vec{f}_{i,1}(z_{0,0},...,z_{i-1,0},z_{0,1},...,z_{i-1,1})) \le z^+_{i+1,1} \\ \vdots \\ z^-_{i+1,j} \le \vec{g}\cdot\vec{f}_{i+1,j}(\vec{g}\cdot\vec{f}_{i,j}(z_{0,0},...,z_{i-1,0},z_{0,1},...,z_{i-1,1},...,z_{0,j-1},...,z_{i-1,j-1})) \le z^+_{i,j} \\ \vdots \end{cases} \qquad (5)$$

The task of finding the parameters of difference operators (2) under condition (4) is the task of solving the interval system of nonlinear algebraic equations (5). It solving algorithm is described in [1].

The considered method is applied for an approximation the surface that describes the distribution of concentrations the some air pollutions in stocky layer of the atmosphere in the central part of Ternopil city in nodes with a given accuracy.

III. Conclusion

The task statement of surface approximation with given accuracy in the nodes applying the difference operator and using methods of interval data analysis is considered.

References

[1] M. Dyvak, "Features of construction the interval system of algebraic equations and the method it's solving in the tasks of identification the interval linear difference operator", Inductive modeling of complex systems, 35-43 pp., 2009 (in Ukrainian).