Coherent Estimation of Probabilistic Characteristics for Periodically Correlated Random Processes In the Case of Preliminary Determination of Period

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Abstract - **The coherent estimators of probabilistic characteristics for periodically correlated random processes in the case of unknown period are analyzed. Shown that these estimators are asymptotically unbiased and consistence.**

Keywords - **Periodically Correlated Random Processes, Mean, Correlations, Consistency, Period of Correlation**

I. INTRODUCTION

In order to use traditional methods for estimation of probabilistic characteristics for periodically correlated random processes (PCRP) – coherent [1], component [2], least squares method [3], linear filtration method [4] the value of period of correlation is required. For period determination may use modified in some way coherent and component statistics. Such functionals Such functionals have extremes at the points, which are close to truth value of period in asymptotic. So the problem of period determination is led to searching for points of extremes for these functionals.

II. PERIOD DETERMINATION

This paper dedicated to analysis of influence of preliminary determination of period on properties of coherent estimator of mean. That estimator in the case of unknown period defines as

$$\hat{m}(t)=\frac{1}{2N+1}\sum_{n=-N}^{N}\int_{-\infty}^{\infty}x\left(t+nx\right)d\left(x-\hat{T}\right)dx\,, \qquad (1)$$

where $d\left(x-\hat{T}\right)$ – delta-function and suppose that period we estimate using functional

$$\hat{m}_l^c=\frac{1}{q}\int_{-q}^{q}x\left(t\right)\cos lw_t t dt\,, \qquad (2)$$

here $w_t=2p/t$, t – test period. Mean of functional (2) has a form

$$S_l=E\hat{m}_l^c=\frac{1}{q}\int_{-q}^{q}m\left(t\right)\cos lw_t t dt\,.$$

Let represent mean as a sum of deterministic and fluctuating parts $\hat{m}_l^c\left(t\right)=S_l\left(t\right)+N_l\left(t\right)$, where $N_l\left(t\right)$ – fluctuating part. The period estimator $\hat{T}$ we search as a extreme point of statistic $\hat{m}_l^c$, i.e. as solution of nonlinear equation:

$$d\tilde{S}_l\left(t\right)/dt+g\,d\tilde{N}\left(t\right)/dt=0\,. \qquad (3)$$

where $\tilde{S}_l\left(t\right)=S_l\left(t\right)/S_l\left(T\right)$, $\tilde{N}_l\left(t\right)=N_l\left(t\right)\Big/\sqrt{E\left[N_l\left(T\right)\right]^2}$ – and $E\left[N_l\left(T\right)\right]^2$ – mean-square value of fluctuating part.

Using small parameter method to (3) compute in the first approximation period value and estimate mean of PRCP:

$$\hat{m}\left(t\right)=\frac{\sum_{n=-N}^{N}\left[x\left(t+nT\right)+A\left[\frac{\partial x\left(t+nx\right)}{\partial x}\right]_{x=T}\int_{-\infty}^{\infty}sx\left(s\right)\sin lw_0 sds\right]}{2N+1}$$

here $A=T\left[2\int_{-q}^{q}sm\left(s\right)\sin lw_0 sds+lw_0\int_{-q}^{q}s^2m\left(s\right)\cos lw_0 sds\right]^{-1}$.

So for finite N the estimator of PCRP's mean in the case of preliminary determination of period of correlations using functional (2) is biased. Its bias and the variance are

$$e\left[\hat{m}\left(t\right)\right]=\frac{A}{2N+1}\sum_{n=-N}^{N}s\sin lw_0 s\left[\frac{\partial b\left(s,t-s+nx\right)}{\partial x}\right]_{x=T}ds$$

The expression for estimator variance is very lengthy and because of it does not given here.

Easy to bring that if condition

$$\int_{-\infty}^{\infty}\left|b\left(t,u\right)\right|du<\infty \qquad (4)$$

is satisfied then $e\left[\hat{m}\left(t\right)\right]\to 0$ at $N\to\infty$.

Similar results are obtained for the estimator of PCRP correlation function.

III. CONCLUSION

In a formula for mean estimator are present additional components, which conditioned by preliminary period determination. It Should be note that bias and variance of mean estimator depend as on mean as on correlation function of PCRP. And if (4) is satisfied that estimator as a estimator of PCRP correlation function are consistent estimators.

REFERENCES

[1] I. Javorskyj, I. Isayev, Z. Zakrzewski, S.P. Brooks (2007) Coherent covariance analysis of periodically correlated random processes, Signal Processing, vol. 87 (1), P. 13-32.

[2] Javorskyj I., I. Isayev, J. Majewski, R. Yuzefovych. (2010) Component covariance analysis for periodically correlated random processes, Signal Processing, vol. 90, P. 1083-1102.

[3] Яворский И.Н., Юзефович Р.М., Кравец И.Б., Закжевски З.(2011) Метод наименьших квадратов при статистическом анализе периодически коррелированных случайных процессов, Известия ВУЗов. Радиофизика, № 1, С. 45-59.

[4] I. Javorskyj, J. Leskow, I. Kravets, E. Gajecka (2011) Linear filtration methods for statistical analysis of periodically correlated random processes. – Part II: Harmonic series representation, Signal Processing, vol. 91, P. 2506-2519.

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