Selection of Sampling Step for Correlation Analysis of Cyclostationary Processes

Roman Yuzefovych

Abstract – **The investigation results of estimators properties for periodically correlated random processes characteristics obtained on the base of discrete time realization are given. Bias and variance formulae for the estimators of mean and covariance Fourier coefficients are found. The conditions for discretization interval when aliasing effects do not occur are obtained. The dependences of estimator characteristics on discretization interval and realization length for modulated signals are analyzed.**

Keywords – **periodically correlated random processes (PCRP); mean; covariance; Fourier coefficients; discrete estimates; bias; variance; aliasing**.

I. Introduction

Discretization is a necessary procedure for signals statistical analysis when engineering tools are used. The most important problem of its implementation is a selection of discretization step. Traditionally, it is selected in correspondence with the highest frequency according to Kotelnikov-Shannon theorem [1]. So the respective lowest frequency of discretization called the Nyquist frequency. At signal statistical analysis the value of discretization step should be selected pursuant to analysis of properties of probabilistic characteristics estimators. The recommendations for its selection can be given only after solving of concrete problems of statistical analysis.

II. The mean Fourier coefficients estimation.

Mean $m(t)=Ex(t)$ of PCRP are periodic functions of time: $m(t)=m(t+T)$. If conditions $\int_0^T |m(t)|dt<\infty$ are satisfied then these characteristics can be represented in the Fourier series form

$$m(t)=\sum_{k\in ¢} m_k e^{ikw_0 t}=m_0+\sum_{k\in ¥}\left(m_k^c\cos kw_0 t+m_k^s\sin kw_0 t\right),$$

here $m_k=\frac{1}{2}\left(m_k^c-im_k^s\right)$, where ¥ – set of natural numbers.

Let put $T=(M+1)h$, where h – sampling time, M – natural number, the realization length is $q=NT$. Coherent estimator of mean [3, 4] at the points $t_n=nh$, $n=\overline{0,M+1}$, is represented as

$$\hat{m}(nh)=\frac{1}{N}\sum_{k=0}^{N-1}x\left[\left(n+k(M+1)\right)h\right].$$

The components estimators can be computed on this base as $\hat{m}_0$, $\hat{m}_l^c$, $\hat{m}_l^s$. The estimators of magnitudes of sine and cosine components for analysis simplification we represent together:

$$\hat{m}_l=\frac{1}{2}\left(\hat{m}_l^c-i\hat{m}_l^s\right)=\frac{1}{K}\sum_{n=0}^{K-1}x(nh)e^{-il\frac{2p}{M+1}n}.$$

The mean of this estimator is:

$$E\hat{m}_l=\frac{1}{K}\sum_{n=0}^{K-1}m(nh)e^{-il\frac{2p}{M+1}n}=\sum_{k\in ¢}m_k f_{k-l}(0,k-1)$$

where

$$f_{k-l}(0,k-1)=\sum_{n=0}^{K-1}e^{i(k-l)\frac{2p}{M+1}n}=d_{k,l+q(M+1)}$$

so $E\hat{m}_l=m_l+\sum_{\substack{q\in ¢\\ q\neq 0}}m_{l+q(M+1)}$. In this case, discretization causes aliasing effect of first kind. It can be avoided if the number of harmonic components is finite. Let that number equals to N_1. The estimator of zeros coefficient $\hat{m}_0$ is unbiased, if $M\geq N_1$. Estimators $\hat{m}_l$ are unbiased, if $M\geq 2N_1$. It means the discretization step should satisfy condition $h\leq T/2N_1+1$.

From above analysis follows the discretization step h should be matched with the number of harmonic components of mean function. Non-execution of request inequalities leads to estimator biasing (aliasing effect of first kind) and appearance of additional components in expressions for estimator variances, which degrade their convergence (aliasing effect of second kind).

III. Conclusion

Discretization leads to increasing of results error obtained at statistical processing of PCRP realization. First of all, it appears as additional components in formulae for variance of mean Fourier coefficients estimators, that is in aliasing. We should separate two types of aliasing. First one leads to estimators bias, which value does not depend on realization length. The second – degrades estimators convergence. Aliasing can be avoided only in the case of PCRP probabilistic characteristics have finite number of harmonic components. The discretization step value h should satisfy some inequalities, which defines by these values.

References

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Roman Yuzefovych – Karpenko Physico-Mechanical Institute of National Academy of Sciences of Ukraine
Naukova str., 5, Lviv, 79601, UKRAINE
E-mail: *abzac@ipm.lviv.ua*