

# Reliability Analysis of Technical Systems without Redundancy with Limited Number of Repairs

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**Abstract** – The object of consideration is the construction of analytical models non-redundant maintainable system with a limited number of repairs. On the basis of the constructed model the analysis of reliability indexes - availability and the probability of downtime is carried out.

**Key words** – reliability, availability, non-redundant system, Markov model, reparability, maintenance, downtime probability.

## INTRODUCTION

Maintainable system widely used in radio electronics and telecommunications, as they allow the replacement or repair of individual items of hardware. The main features of such systems is the large number of possible states and the dependence of the reliability on such factors as the failure rate of individual elements, the intensity of repair, strategy maintenance, etc.

Therefore, analysis of the reliability of maintainable systems are more difficult task compared with the analysis of the reliability of non-maintainable systems. Methods of reliability analysis depends on the type of the distribution of non-failure operating time of elements and their recovery time. Thus, when the exponential distribution using we can obtain analytical expressions for calculation of reliability indices, and in case using of non-exponential distribution we must use a numerical methods in combination with the methods of statistical simulation (Monte Carlo), which require high computational burden. It should be noted that with increasing the reliability of the element base the computational burden are increase, which reduces the effectiveness of simulation and in many cases it becomes almost impossible to implement. Therefore, analyzing the reliability by analytical methods has not lost relevance in our time. Results of calculating of the reliability of maintainable systems by analytical methods is known [1, 2] for the following cases:

- Maintainable system with  $n$  different elements of a permanent redundancy, which is maintained by  $n$  repair teams (unlimited maintenance);
- Maintainable system with  $n$  identical elements and one repair teams (limited maintenance).

Among the large number of known methods of calculating the reliability should be allocated the methods based on the

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theory of Markov processes that represent a mathematical apparatus, suitable to describe the evolution of the system (transition from state to state) in continuous time. Their advantage is the relative ease of construction Chapman-Kolmogorov equations, the possibility of obtaining analytical expressions for the reliability of systems and the possibility of reduction the non-Markov type processes to Markov processes

## BUILDING THE ANALITICAL RELIABILITY MODEL

In this paper an analytical model built reliability of the non-redundant maintainable system with a limited number of repairs and analyzed indices of reliability.

Structure of system includes two devices, which are interconnected sequentially in the sense of reliability (basic connection), ie the system are operable only when both devices are working. In case of refusal of any device system becomes non-operable, and a repair team restores the performance device that refused, then the whole system becomes operational. Each device can be renewed only once. The ultimate failure of the system comes, when repeated refusal of any device occurs. Parameters of reliability of system are:  $\lambda_1$  and  $\lambda_2$  – failure rate of the first and second device respectively;  $\mu_1$  and  $\mu_2$  – intensity of repair of the first and second device respectively.

The matrix of states of the system has the form:

№ device	Number or state								
	0	1	2	3	4	5	6	7	8
1	P	R	P	P1	P	P1	R	P1	F
2	P	P	R	P	P1	R	P1	P1	F
System	P	R	R	P	P	R	R	P	F

P - an operation state; R - state of repair; P1 - an operation state of after the first repair; F - faulted state.

The time dependence of the probability of staying in the relevant states describing the Chapman-Kolmogorov equation:

$$\frac{dP_0(t)}{dt} = -(I_1 + I_2)P_0(t)$$

$$\frac{dP_1(t)}{dt} = I_1P_0(t) - m_1P_1(t)$$

$$\frac{dP_2(t)}{dt} = I_2P_0(t) - m_2P_2(t)$$

$$\frac{dP_3(t)}{dt} = m_1P_1(t) - (I_1 + I_2)P_3(t)$$

$$\begin{aligned} \frac{dP_4(t)}{dt} &= m_2 P_2(t) - (I_1 + I_2) P_4(t) \\ \frac{dP_5(t)}{dt} &= I_2 P_3(t) - m_2 P_5(t) \\ \frac{dP_6(t)}{dt} &= I_1 P_4(t) - m_1 P_6(t) \\ \frac{dP_7(t)}{dt} &= m_2 P_5(t) + m_1 P_6(t) - (I_1 + I_2) P_7(t) \\ \frac{dP_8(t)}{dt} &= I_1 \cdot P_3(t) + I_2 \cdot P_4(t) + (I_1 + I_2) \cdot P_7(t) \end{aligned}$$

The initial state of the system at  $t = 0$ :  $P_0(0) = 1$ ;  $P_1(0) = P_2(0) = P_3(0) = P_4(0) = P_5(0) = P_6(0) = P_7(0) = P_8(0) = 0$ . For solution of Chapman-Kolmogorov equation the Laplace transformation is advisable to using. The Laplace transformation allows to transit from the differential equations to algebraic equations, which recorded in relation to operator images of probabilities. Thus solving it, the time dependence of probabilities is got by using inverse Laplace transform. These operations are performed using the symbolic mathematics (eg, MATLAB or MATHCAD). The time dependence of the relevant probabilities is carried out as result.

**RELIABILITY ANALYSIS NON-REDUNDANT SYSTEM WITH DOWNTIME**

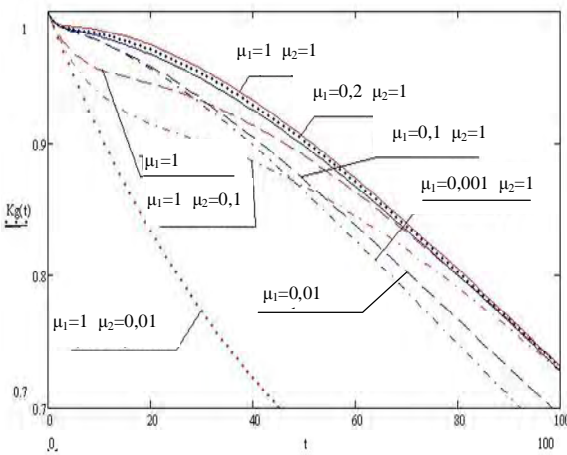


Fig. 1. A time dependence of availability for different values of the rate of repair

The presence of time dependence of the probabilities of staying in certain states allows determine basic indexes of reliability. Thus, the function of availability (Fig. 1) system is:

$$Kg(t) = P_0(t) + P_3(t) + P_4(t) + P_7(t)$$

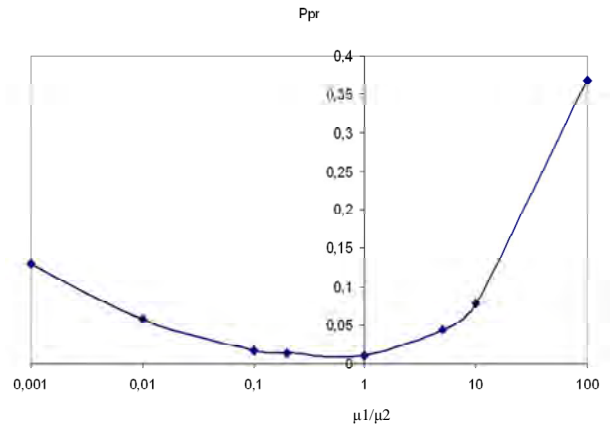


Figure 2. Dependence of the maximum value function of downtime for different the ratios of intensities of repair

The probability of downtime (Fig. 2) due to repair:

$$Ppr(t) = P_1(t) + P_2(t) + P_5(t) + P_6(t)$$

The function of availability and the probability of downtime is determined with such the input data: failure rate of 1 device  $\lambda_1 = 1 \cdot 10^{-3}$ , failure rate of the 2nd device  $\lambda_2 = 1 \cdot 10^{-2}$ , rate of repair takes such values:

$\mu_1$	1	1	1	1	0,2	0,1	0,01	0,001
$\mu_2$	1	0,2	0,1	0,01	1	1	1	1

**CONCLUSION**

The developed analytical model reliability of non-redundant maintainable system with a limited number of repair allows to carry out multivariate analysis of reliability indices to select indexes maintenance (rate of the repair or maintenance) by the criterion of minimum probability of downtime  $Ppr(t)$  of system. Minimum probability of downtime is provided by reduction in repair time of less reliable system.

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