Linear Filtration Methods for Statistical Analysis of Periodically Correlated Random Processes

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Abstract - **Coherent and component methods for mean and covariance function estimation are analyzed using linear filtration theory.**

Keywords - **Cyclostationary Processes, Coherent Method, Component Method, Linear Filtration Methods.**

I. INTRODUCTION

Periodically correlated (PC) random processes are popular models of physical phenomena with cyclic and stochastic behavior [1, 2]. They are also known as cyclostationary or periodic stationary random processes. This means in particular that mean function of real-valued process $m(t) = Ex(t) = m(t+T)$, covariation function $b(t,u) = E\bar{x}(t)\bar{x}(t+T,u) = b(t+T,u), \bar{x}(t) = x(t) - m(t)$. Also we may use the following Fourier series representation

$$m(t) = \sum_{k\in ¢} m_k e^{ikw_0 t},\ \ b(t,u) = \sum_{k\in ¢} B_k(u) e^{ikw_0 t}.$$

Traditional methods for PC processes statistical analysis are coherent and component methods. The coherent method in estimating first and second order characteristics of PC signals are based on synchronous averaging:

$$\hat{m}(t) = \frac{1}{N}\sum_{n=0}^{N-1} x(t+nT),$$

$$\hat{b}(t,u) = \frac{1}{N}\sum_{n=0}^{N-1}[x(t+nT) - \hat{m}(t+nT)]\times$$
$$\times[x(t+u+nT) - \hat{m}(t+u+nT)],$$

here N is a number of periods, that are averaged, hereby the length of realization fragment $q = NT + u_m$, where u_m is the autocovariance function maximal time lag. Component estimates are built like trigonometric polynomial:

$$\hat{m}(t) = \sum_{k=-N_1}^{N_1} \hat{m}_k e^{ikw_0 t},\ \ \hat{b}(t,u) = \sum_{k=-N_2}^{N_2} \hat{B}_k(u) e^{ikw_0 t},$$

where

$$\hat{m}_k = \frac{1}{q}\int_0^q x(t) e^{-ikw_0 t} dt,$$

$$\hat{B}_k(u) = \frac{1}{q}\int_0^q [x(t) - \hat{m}(t)][x(t+u) - \hat{m}(t+u)] e^{-ikw_0 t} dt,$$

More detailed analysis of coherent and component estimates can be found in [1]. In this paper we analyze these methods from the point of view of linear filtration theory.

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II. LINEAR FILTRATION IN PC SIGNALS

Coherent and component methods for probability characteristic estimation of PC process can be represented as particular cases of demodulation method [1], there for the estimator $\hat{m}(t)$ we have:

$$\hat{m}(t) = \int_{t_0}^{t} x(t)h(t-t)dt = \int_0^{t-t_0} x(t-t)h(t)dt,$$

where $h(t)$ is the weight function. Here we suppose that signal is given in the interval $[t_0, t]$ and its length $q = t - t_0 = NT$.

If the weight function $h(t)$ is taken in following form:

$$h(t) = \frac{1}{N}\sum_{n=0}^{N-1} d(t - nT),$$

then we obtain a coherent estimate:

$$\hat{m}(t) = \frac{1}{N}\sum_{n=0}^{N-1}\int_0^{t-t_0} x(t-t)d(t-nT)dt = \frac{1}{N}\sum_{n=0}^{N-1} x(t-nT).$$

When weight function has a form

$$h(t) = \frac{1}{t-t_0}\sum_{k=-N_1}^{N_1} e^{ikw_0 t},$$

we obtain the component estimate:

$$\hat{m}(t) = \int_0^{t-t_0} x(t-t)[\frac{1}{t-t_0}\sum_{k=-N_1}^{N_1} e^{ikw_0 t}]dt =$$
$$= \sum_{k=-N_1}^{N_1} e^{ikw_0 t}[\frac{1}{t-t_0}\int_{t_0}^{t} x(t) e^{ikw_0 t} dt].$$

Transfer function of the filter in the case of coherent estimate has the form

$$H(w) = \frac{1}{N}\sum_{n=0}^{N-1} e^{-iwnT} = e^{i\frac{wT}{2}}(N-1)\frac{\sin[\frac{NwT}{2}]}{N\sin[\frac{wT}{2}]},$$

and in the case of component estimate

$$H(w) = \sum_{k=-N_1}^{N_1} e^{-i(w-kw_0)\frac{q}{2}} \sin[(w-kw_0)\frac{q}{2}] \Big/ (w-kw_0)\frac{q}{2}.$$

III. CONCLUSION

General approach for PC process estimation theory can be developed on the base of linear filtration methods.

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