Radar Signals Classification

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Abstract: - **The paper presents on the base of theoretical approach of time-frequency analysis and time-frequency representation optimizations, classification results of real radar signals. The results show, that applying class-dependent methodology it is possible to achieve acceptable classification level on the base of few intercepted pulses.**

Keywords – **Radar signal, time-frequency analysis, signals classification.**

I. INTRODUCTION

The distinctive features of modern radar signal are hidden in its time-frequency structure due to radar waveforms complexity. Time-frequency distribution concept offers a new approach in radar signals classification/identification. It is expected, that due to unintentional modulation within the pulse it will be possible to distinguish among radar emitters on the base of short record of collected data. The majority of identification techniques require a priori information on time and/or frequency parameters of signals being analyzed. Instead of assuming a priori frequency or time analysis time-frequency representation optimization [1-6] gives an opportunity directly on the data (signal) coming from emitters. Such an approach includes kernel design that optimizes differences between primarily defined classes [1-3]. Designed function modifies time-frequency representation of a signal. The paper presents some results of radar signals classification on the base of time-frequency representation optimization.

II. WIGNER DISTRIBUTION AND AMBIGUITY FUNCTION

The Wigner distribution of a signal *f(t)* is given by the equation (1) [5]

$$W_f(t,w) = \int_{-\infty}^{+\infty} e^{-jwt} f(t+t/2) f^*(t-t/2) dt \qquad (1)$$

where: w - angular frequency, * - indicates complex conjugation, t- relative time. Signal spectrum is *F(w)*. With respect to the last equation the Wigner distribution is regarded as the spectrum of the signal $f(t+t/2)f^*(t-t/2)$ considered as a function of t with t as a fixed parameter. Thus the inverse Fourier transform yields

$$f(t+t/2) f^*(t-t/2) = \frac{1}{2p} \int_{-\infty}^{+\infty} e^{jwt} W_f(t,w) dw \qquad (2)$$

that using substitutions $t_1=t+t/2$ and $t_2=t-t/2$ for the case $t_1=t_2=t$ gives

$$f(t) f^*(t) = \left| f(t) \right|^2 = \frac{1}{2p} \int_{-\infty}^{+\infty} W_f(t,w) dw \qquad (3)$$

The integral of the WD over frequency variable at a certain time t yields instantaneous signal power at that time. Integration of equation (3) with respect to time interval $t_a<t<t_b$ yields

$$\frac{1}{2p} \int_{t_a}^{t_b} \left[\int_{-\infty}^{+\infty} W_f(t,w) dw \right] dt = \int_{-t_a}^{t_b} \left| f(t) \right|^2 dt \qquad (4)$$

The last relation shows, that the integral of the WD of the signal *f(t)* over the infinite strip $-¥<w<+¥$ and limited time internal $t_a<t<t_b$ is equal energy contained in *f(t)* in analysed time interval. Wigner distribution and ambiguity function (AF) look very much alike. They are however different representations of a signal represented by the following equation[6].

$$A_f(x,t) = \frac{1}{2p} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-j(xt-wt)} W_f(t,w) dt dw \qquad (5)$$

Last equation shows that ambiguity function and Wigner distribution are related by the two-dimensional Fourier transform. Ambiguity function can be used in kernel function design for classification.

III. CLASS-DEPENDENT TIME-FREQUENCY REPRESENTATION

A generalized class of time-frequency representation of a signal was introduced by Cohen [4,5]. The class is given by

$$C_f(t,w;\Phi) =$$

$$\frac{1}{2p} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j(xt-tw-xu)} \Phi(x,t) f(u+t/2) f^*(u-t/2) du dx dt \qquad (6)$$

where x and t denotes frequency and lag respectively, Φ is an arbitrary kernel function, determining the particular representation in the class. Using the AF definition of *f*

$$A_f(x,t) = \int_{-\infty}^{\infty} e^{-jxu} f(u+t/2) f^*(u-t/2) du \qquad (7)$$

one rewrite equation (1) as

$$C_f(t,w;\Phi) = \frac{1}{2p} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j(xt-tw)} \Phi(x,t) A_f(x,t) dx dt \qquad (8)$$

or in terms of Wigner-Ville distribution as

$$C_f(t,w;\Phi) = \frac{1}{2p} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} j(t-t,w-x) WV(t,x) dt dx \qquad (9)$$

where WV denotes Wigner-Ville distribution and

$$j(t,w) = \frac{1}{2p} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j(xt-tw)} \Phi(x,t) dx dt \qquad (10)$$

The interpretation of equation (9) is that that all members of this class of signal representation can be obtained by transformation of *WV* characterized by the kernel *j(t,w)* that is related to kernel *F(x,t)* by a two-dimensional Fourier transformation accordingly to equation (10). The equation (12) shows that all time-frequency representation can be obtained by the use of appropriate kernel and two-dimensional transformation of signal ambiguity function [6].

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Signal ambiguity plain presents some interesting properties for classification. It is determined as a Fourier transform of signal autocorrelation function. An individual position on ambiguity plain determines information on time-frequency structure of a signal. The issue of problem is included in appropriate design of kernel. When the kernel $F(x,t)$ is designed with purpose of classification, it is called as the signal class-dependent kernel [2]. In short form it is called class-dependent kernel and is denoted as $F_{CD}(x,t)$. The corresponding time-frequency representation is associated with that kernel and is called as a class-dependent time-frequency representation. This representation is given by

$$C_{CD}[x,t] = F_x^{-1}\{F_t\ \{\Phi_{CD}[x,t]A_f[x,t]\}\} \tag{11}$$

Thus it has become possible to design class-dependent time-frequency representation and observe time-frequency signal structure being under classification. Except representation given by the equation (6) it is possible to use another ones that could applied as base representations. To do this it is necessary to design multiplicative transformation of kernel $F_T(x,t)$, that transforms current time-frequency representation into "new" one. Since prior class-dependent kernel has been used to transformation, "new" class-dependent kernel $F_{CD}'(x,t)$ will be given by

$$\Phi_{CD'}[x,t] = \Phi_{CD}[x,t]/\Phi_T[x,t] \tag{12}$$

The associated class-dependent time-frequency distribution is given by

$$C_{CD}[x,t] = F_x^{-1}\{F_t\ \{f_{CD}[x,t](f_T[x,t]A_f[x,t])\}\} \tag{13}$$

The criterion for time-frequency representation, resulting, from equation (8) is

$$\left|\Phi_T[x,t]\right| \neq 0 \tag{14}$$

for all (x,t) values, preserving all information in original ambiguity function [2,8].

IV. 2. Radar signals classification

The approach presented above has been applied to radar signals classification. The examination includes stage of training and classification. During training four radar pulses are acquired and then ambiguity function is calculated that is followed by Fisher's discriminant ratio [2] and finally kernel function is designed. For each group of pulses coming from particular emitter I pulses are taken to the training. Than autocorrelation function and ambiguity function is calculated. After calculation of ambiguity function for each training examples class average values of ambiguity functions are determined:

$$\overline{\mathbf{A}}^{(c)} = \frac{1}{I}\sum_{i=1}^{I}\mathbf{A}_{y_i^{(c)}} \tag{15}$$

where: I – number of examples creating class, $A_{yi}^{(c)}$ – ambiguity function of i-th example for particular class. Incoming kernel function should be designed in the manner providing distance maximization between calculated ambiguity functions. It is made by ordering the values of kernel function accordingly to Fisher's discriminant:

$$FDR[x,t] = \frac{\sum_{i=1}^{C}\sum_{j=1}^{C}\left|\overline{A}^{(i)}[x,t] - \overline{A}^{(j)}[x,t]\right|^2}{\sum_{i=1}^{C}\left(s^{(c)}[x,t]\right)^2} \tag{16}$$

where $s(c)$ – estimated standard deviation in the point (x,t) of ambiguity plane.

The process of emitters classification is based on the square discrimination function [2]. Classification algorithm declares x observation to given class C, when i index satisfies to the following condition:

$$i = \underset{i=1,\dots,C}{\arg\min}\, d(C_x^f, \overline{C}_i^f) \tag{17}$$

where d - distance between classes,

C_x^f - time-frequency representation of signal under classification, $\overline{C}_i^f$ - average form of signals taken to the training of C class,

The classification formula is given by the equation:

$$l_c(f_{CD}\,\mathbf{o}\,\mathbf{A}_x) = \left(f_{CD}\,\mathbf{o}\,\mathbf{A}_x - f_{CD}\,\mathbf{o}\,\overline{\mathbf{A}}^{(c)}\right)^T \cdot \left\|\Sigma_c^{-1}\right\|_F \left(f_{CD}\,\mathbf{o}\,\mathbf{A}_x - f_{CD}\,\mathbf{o}\,\overline{\mathbf{A}}^{(c)}\right) + \ln\left|\Sigma_c\right| \tag{18}$$

where $\mathbf{A_x}$ – ambiguity function of a signal under classification,

$\sum_c$ - covarinace matrix of class calculated on the training stage,

$\|.\|_F$ - Frobenius norm of matrix,

$|.|$ - matrix determinant.

Signal of unknown x emitter is declared to the given class when Frobenius norm of square matrix form of discrimination function has minimum value:

$$\underset{C=1,\dots,n}{\arg\min}\left\|l_c(f_{CD}\,\mathbf{o}\,\mathbf{A}_x)\right\|_F \Rightarrow x \in C \tag{19}$$

Last equation ends the classification process.

V. Conclusions

Classification method has been tested on real signals. The effectiveness of 83% and 67% respectively for unmodulated and linear frequency modulated signals is received. On the base of performed examinations one can say that classification results show acceptable level of effectiveness however it is necessary extend examinations on wider class of radar emitters

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