Frequency Model of Linear Parametric Circuit in Form of Matrix L.A.Zadeh's Equation

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Abstract – **In this paper matrix frequency model of linear parametric circuit based on L.A.Zadeh's equation is offered.**

Keywords – **linear parametric circuits, frequency models, frequency-symbolic method.**

I. INTRODUCTION

It is known [1] that differential equation which links input and output of linear parametric circuit in time domain

$$a_n(t)y^{(n)} + a_{n-1}(t)y^{(n-1)} + \ldots + a_0(t)y = b_m(t)x^{(m)} + \\ + b_{m-1}(t)x^{(m-1)} + \ldots + b_0(t)x\,, \qquad (1)$$

where y – output and x – input variables, t – independent variable – time, $a_i(t)$, $b_j(t)$ – known functions of the time ($n \geq m$ – order of the equation), can be converted into the complex domain of variable $s = j\omega$ in the next form:

$$\frac{1}{n!}\cdot\frac{d^n A(s,t)}{ds^n}\cdot\frac{d^n W(s,t)}{dt^n} + \mathbf{L} + \frac{dA(s,t)}{ds}\cdot\frac{dW(s,t)}{dt} + \\ + A(s,t)\cdot W(s,t) = B(s,t)\,, \qquad (2)$$

where $W(s,t)=Y(s,t)/X(s)$ - conjugate parametric transfer function of the circuit, $A(s,t) = a_n(t)s^n + a_{n-1}(t)s^{n-1} + \ldots + a_0(t)$, $B(s,t)=b_m(t)s^m+b_{m-1}(t)s^{m-1}+\ldots+b_0(t)$; $a_i(t)$, $b_j(t)$ – appropriate coefficients of the equation (1); $Y(s,t)$, $X(s)$ – images of the output $y(t)$ and the input $x(t)$ variables in the time domain respectively. The equation (2) is called L.A.Zadeh's equation. In this work is proposed an analogous conversion of the linear differential equation system (LDES) that describes given circuit into frequency domain.

II. MAIN PART

The linear parametric circuit is described in form of the next LDES in operator form:

$$A(p,t)\times Y = B(p,t)\times X\,, \qquad (3)$$

where p – operator that only means differentiation act d/dt; $Y=[y_1(t), y_2(t),\ldots, y_r(t)]^t$ – r-dimensional vector of unknown variables of the parametric circuit; $A(p,t)=[\{A_{ij}(p,t)\}]$ – ($r\times r$)-dimensional matrix of the operator differential expressions $A_{ij}(p,t) = a_{ij,n}(t)p^n+a_{ij,n-1}(t)\cdot \cdot p^{n-1}+a_{ij,0}(t)$; $a_{ij,n}(t), a_{ij,n-1}(t), a_{ij,0}(t)$ – known real functions of time t; $B(p,t)=[\{B_{ii}(p,t)\}]$ - ($r\times r$)-dimensional diagonal matrix of the operator differential expressions $B_{ii}(p,t)=b_{ii,m}(t)p^m+b_{ii,m-1}(t)p^{m-1}+b_{ii,0}(t)$; $b_{ii,m}(t)$, $b_{ii,m-1}(t)$, $b_{ii,0}(t)$ – known real functions of time t; $X=[x_1(t), x_2(t),\ldots, x_r(t)]^t$ – r-dimensional vector of input signals $x_1(t), x_2(t),\ldots, x_r(t)$; $i,j=1,2,\ldots,r$; r,n,m – positive integers, $0\leq n\leq 2$, $0\leq m\leq 2$.

In consequence of linearity of equations (1) and (3), conversion of equation (3) into frequency domain is as well lawful and leads to formation of the matrix frequency circuit model that also has form of equation (2)

$$\frac{1}{n!}\cdot\frac{d^n A(s,t)}{ds^n}\times\frac{d^n W(s,t)}{dt^n} + \mathbf{L} + \frac{dA(s,t)}{ds}\times\frac{dW(s,t)}{dt} + \\ + A(s,t)\times W(s,t) = B(s,t)\,, \qquad (4)$$

but here $A(s,t)$ – matrix received from matrix $A(p,t)$ by changing variable p with variable s; $W(s,t)=[\{W_{ij}(s,t)\}]$ – ($r\times r$)-dimensional unknown matrix of parametric transfer functions, $W_{ij}(s,t) = Y_i(s,t)/X_j(s)$ – parametric transfer function from input signal $X_j(s)$ into variable $Y_i(s,t)$ of the linear parametric circuit in frequency domain s; $B(s,t)$- diagonal matrix formed from matrix $B(p,t)$ by replacing variable p with variable s. The number of unknown variables in system (3) equals r, thus in matrix frequency model (4) its number is r^2. Determined from matrix frequency model (4) parametric transfer functions $W_{ij}(s,t)$ with input signals $X_1(s_1) = e^{s_1 t}$, $X_2(s_2) = e^{s_2 t}$, … , $X_r(s_r) = e^{s_r t}$, define output signals $y_i(t)$ in the next form

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ \mathbf{L} \\ y_r(t) \end{bmatrix} = \mathrm{Re}\left\{\begin{bmatrix} W_{11}(s,t) & W_{12}(s,t) & \mathbf{L} & W_{1r}(s,t) \\ W_{21}(s,t) & W_{22}(s,t) & \mathbf{L} & W_{2r}(s,t) \\ \mathbf{L} & \mathbf{L} & \mathbf{L} & \mathbf{L} \\ W_{r1}(s,t) & W_{r2}(s,t) & \mathbf{L} & W_{rr}(s,t) \end{bmatrix} \times \begin{bmatrix} e^{s_1 t} \\ e^{s_2 t} \\ \mathbf{L} \\ e^{s_r t} \end{bmatrix}\right\}. \qquad (5)$$

In this work for the test parametric circuit that is described with LDES by node voltages method the matrix frequency model in form of equation (4) is built. That equation is solved by frequency symbolic method (FS-method)[2]. There are also determined time dependences for nodal voltages of this circuit using expression (5).

III. CONCLUSION

Formed matrix frequency model (4) of linear parametric circuit is frequency analog of the LDES (3) and fully describes the circuit in frequency domain. In particular: a) solution of the system (4) is parametric transfer functions matrix $W_{ij}(s,t)$ that we form by FS-method which provides sufficient accuracy; b) parametric transfer functions with preset harmonic input signals define required circuit variables in time domain using expression (5).

REFERENCES

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