Characteristics for Reliability Estimation of Hierarchical Systems

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Abstract – **Main reliability characteristics for unrestorable symmetric hierarchical systems ramified to level 2 and with ageing output elements are examined in this paper. Models for the failure probability and the average duration of the system's stay in the prescribed availability condition are worked out in the case when the lifetime of ageing output elements is circumscribed by the Weibull distribution.**´

Keywords – **hierarchical systems, symmetric systems, reliability characteristics.**

I. INTRODUCTION

There has developed the intensive study of distribution types in the reliability theory [1]. When the hazard function ***h(t)*** is a constant, the corresponding exponential distribution is one of the most important failure law in reliability work, particularly in waiting-time problems [2]. The Weibull distribution is valid for failures arising as a result of exhaustion and ageing [3].

II. ESTIMATION OF RELIABILITY CHARACTERISTICS

Expressions for probability distribution calculation of count of operating output elements of the system are deduced on the basis of the generation function (2). By analogy with [4] an expression for the probability distribution calculation of count of operating output elements is

$$P_{2W}(x_2,t)=e^{-l_0 t}\sum_{x_1=ceil\left(\frac{x_2}{a_2}\right)}^{a_1} C_{a_1}^{x_1} e^{-l_1 x_1 t}\times \tag{1}$$

$$\times(1-e^{-l_1 t})^{a_1-x_1} C_{a_2 x_1}^{x_2} e^{-l_2 x_2 t^{b_2}}(1-e^{-l_2 t^{b_2}})^{a_2 x_1-x_2}$$

where k is the system availability condition (not less then k operating output elements, $0<k£a_1a_2$).

We use $T_{2W}(x_2)$ to denote the average duration of the system's stay in a state of x_2 operating output elements on condition that lifetime of ageing output elements is circumscribed by the Weibull distribution.

Under condition $0<x_2\le N_2$ we obtain the following expression:

$$T_{2W}(x_2)=\sum_{x_1=ceil\left(\frac{x_2}{a_2}\right)}^{a_1} C_{a_1}^{x_1} C_{a_2x_1}^{x_2}\sum_{j_1=0}^{a_1-x_1} C_{a_1-x_1}^{j_1}(-1)^{j_1}\times \tag{2}$$

$$\times\sum_{j_2=0}^{a_2x_1-x_2} C_{a_2x_1-x_2}^{j_2}(-1)^{j_2}\int_0^{\infty} e^{-(l_0+l_1(x_1+j_1))t} e^{-l_2(x_2+j_2)t^{b^2}}\,dt.$$

Notice that $T_{2W}(0)=\mu$.

The system availability condition is that there are not less than k operating output elements of the system ($0<k£N_2$). The sum of average durations of the system's stay in states over count of output elements from k до N_2 is equal to the average duration of the system's stay in the prescribed availability condition k.

Let $T_{\Gamma 2W}(k)$ be the average duration of the system's stay in the availability condition k provided that lifetime of ageing output elements is circumscribed by the Weibul distribution. We obtain:

$$T_{\Gamma 2W}(k)=\sum_{x_2=k}^{a_1a_2} T_{2W}(x_2)=\sum_{x_2=k}^{a_1a_2}\sum_{x_1=ceil\left(\frac{x_2}{a_2}\right)}^{a_1} C_{a_1}^{x_1} C_{a_2x_1}^{x_2}\sum_{j_1=0}^{a_1-x_1} C_{a_1-x_1}^{j_1}(-1)^{j_1}\times \tag{3}$$

$$\times\sum_{j_2=0}^{a_2x_1-x_2} C_{a_2x_1-x_2}^{j_2}(-1)^{j_2}\int_0^{\infty} e^{-(l_0+l_1(x_1+j_1))t} e^{-l_2(x_2+j_2)t^{b_2}}\,dt.$$

CONCLUSIONS

The paper deals with mathematical models of main time reliability characteristics for unrestorable hierarchical systems.

It is necessary to work out methods of reliability prediction with regard for systems' specific features such as possibility of structure rearrangement, preservation of serviceability in case of partial failures at the expense structural redundancy.

Thus, expressions are worked out for evaluation of two main time reliability characteristics of composite electropower systems:

– the average duration of the system's stay in a state of x_2 operating output elements;

– the average duration of the system's stay in the prescribed availability condition.

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