Scattering of Electromagnetic Waves by Thin Cylinder: Asymptotic Solution

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Abstract – **Electromagnetic wave scattering by infinite cylinder is investigated for the case when the radius of cylinder tends to zero. The theory is developed under the assumptions $ka << 1$, where a is the radius of cylinder and k is the wavenumber. The results of numerical simulation show good agreement with the theory. They open a way to numerical simulation of procedure for creating materials with prtescribed refraction coefficient.**

Keywords – **Diffraction Problem, Thin Cylinder, Asymptotic Solution, Numerical Simulation.**

I. INTRODUCTION

Theory of wave scattering by small particles of arbitrary shapes was developed by A. G. Ramm in [1]-[2]. This approach was generalized to electromagnetic (EM) wave scattering by set of parallel cylinders [3]. The peculiarity of approach proposed there is the following:

i) The cylinder is of small radius a, and value $ka << 1$, where k is the wavenumber of media outside of cylinder.

ii) The solution to the wave scattering problem is considered at the limit $a \to 0$.

iii) The approach proposed allows generalization to the case of many cylinders that open the possibility to change the refraction coefficient in limiting medium.

II. SOLUTION OF SCATTERING PROBLEM FOR THIN CYLINDER

Let D_m, $1 \le m \le M$ is a set of non-intersecting domains in xOy plane. Let $\hat{x}_m \in D_m$, $\hat{x}_m = (x_{m1}, x_{m2})$ is a point inside D_m and C_m is the cylinder with cross-section D_m and the axis parallel to z-axis, passing by $\hat{x}_m$. We suppose that $\hat{x}_m$ is the center of the disc D_m, and D_m is a disc of radius a, and $a = 0.5 \mathrm{Diam} D_m$.

The EM wave scattering problem consists in determination of solution to the Maxwell equations

$$\nabla \times E = i\omega\mu H, \tag{1}$$

$$\nabla \times H = -i\omega\varepsilon E \tag{2}$$

in C', such that $E_t = 0$ on ∂C, where ∂C is the union of surfaces of cylinders C_m, C' is complement of all C_m, E_t is the tangential component of E. The values μ and ε are the constants, ω is the frequency, $k^2 = \omega^2 \varepsilon\mu$. The refraction coefficient n_0^2 of initial medium is determined as $n_0^2 = \varepsilon\mu$, that

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implies $k^2 = \omega^2 n_0^2$. The solution to problem (1)-(2) has the form [3]

$$E(x) = E_0(x) + v(x),$$

$$x = (x_1, x_2, x_3) = (x, y, z) = (\hat{x}, z), \tag{3}$$

where $E_0(x)$ is the incident field, and v is the scattered field which satisfies the radiation condition

$$\sqrt{r}(\frac{\partial v}{\partial r} - ikv) = o(1), \quad r = (x_1^2 + x_2^2)^{1/2}. \tag{4}$$

For this case we assume that

$$E_0(x) = \exp(i\kappa y + ik_3 z)e_1, \quad \kappa^2 + k_3^2 = k^2, \tag{5}$$

where $\{e_j\}, j = 1, 2, 3$ are the unit vectors of the Cartesian coordinate system.

We consider here the case of $E-$ waves, or $TH-$ waves, namely $H_3 := H_z = 0$ that produces the formulas [3]

$$E = \sum_{j=1}^{3} E_j e_j, \quad H = H_1 e_1 + H_2 e_2 = \frac{\nabla \times E}{i\omega\mu}. \tag{6}$$

The components of electrical field E and H can be expressed by formulas [3]

$$E_j = \frac{ik_3}{\kappa^2} u_{x_j} e^{ik_3 z}, \quad j = 1, 2, \quad E_3 = u e^{ik_3 z}, \tag{7a}$$

$$H_j = \frac{i\omega\varepsilon}{\kappa^2} u_{x_j} e^{ik_3 z}, \quad j = 1, 2, \quad H_3 = 0, \tag{7b}$$

where the function u solves the respective two-dimensional boundary problem [3]. Formulas (7) give the total representation of the EM field in the presence of thin cylinder. We derive the specific formula for the EM field components in the case when the radius a of cylinder tends to zero.

III. ASYMPTOTIC OF EM WAVE SCATTERING BY ONE CYLINDER

The solution $u(\hat{x})$ of the above mentioned boundary problem is found in the form [3]:

$$u = e^{iky} + \int_{S_1} g(\hat{x}, t)\sigma(t)dt,$$

$$g(\hat{x}, t) = \frac{i}{4} H_0^{(1)}(\kappa |\hat{x} - t|), \tag{8}$$

where S_1 is the boundary of one cylinder D_1, $H_0^{(1)}$ is the Hankel function of order 1, with index 0, and function s is found from the boundary condition of the boundary problem.

We use the following representation for function $g(kr)$ [1]:

$$g(\kappa r) = \alpha(\kappa) + \frac{1}{2\pi}\ln\frac{1}{r} + o(1) \text{ , as } r \to 0, \qquad (9)$$

where

$$\alpha(\kappa) = \frac{i}{4} - \frac{1}{2\pi}\ln\frac{2}{\kappa} + \frac{\gamma}{2\pi}. \qquad (10)$$

and γ is Euler's constant, $\gamma = -\Gamma'(1) = 0.5772...$, $\Gamma(z)$ is the gamma-function.

Using the formulas (9)-(10) for function $g(\kappa r)$, we can reduce the finding the solution (8) to the calculation of integral

$$Q = \int_{S_1} \sigma(t)dt. \qquad (11)$$

After series of transformations of (11), function u can be presented as

$$u(\mathbf{\mathcal{X}}) = u_0(\mathbf{\mathcal{X}}) - \frac{2\pi}{\ln(1/a)} g(\mathbf{\mathcal{X}}0) + o(1),$$

$$a \to 0, \; |\mathbf{\mathcal{X}}| > a, \qquad (12)$$

where $u_0(x) = e^{i\kappa x_2}$, $x_2 = y$.

In conclusion, EM field, scattered by the single cylinder, is calculated by formulas (7) taking into account formula (12).

VI. NUMERICAL SIMULATION

The numerical approach to solve the acoustic wave scattering problem was developed in [4] and generalized for EM wave scattering in [5]. Some part of algorithms from [4] and [5] is used for numerical simulation here. The results of numerical calculations are shown for the various values of the radius a of cylinder. The resulting field approaches to the field of thin wire when the radius a of cylinder tends to zero. It appears that the value of the parameter k_3 has an influence on the character of scattered field in the considerable extent. In the Figs. 1-3, the amplitude of component E_3 of electrical field is shown for the various values of k_3. One can see that amplitude becomes asymmetry if the value of k_3 increase.
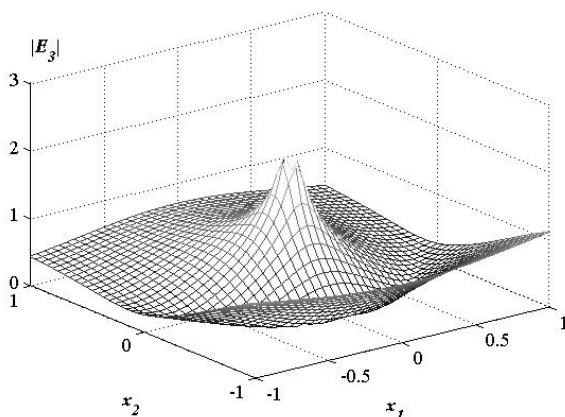
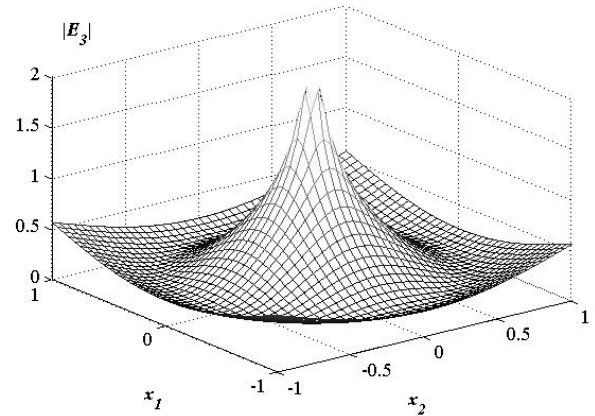Fig. 1. Amplitude E_3 for $k_3 = 0.5$

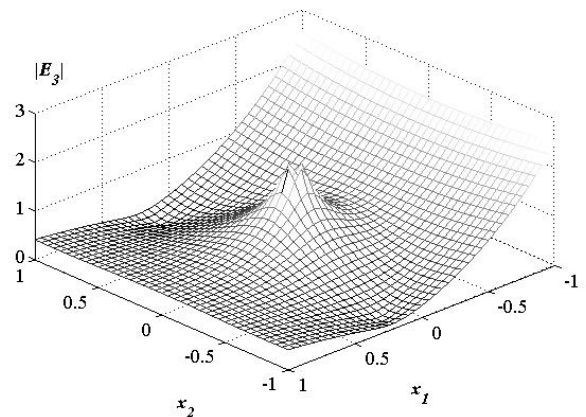Fig. 2. Amplitude E_3 for $k_3 = 1.0$

Fig. 3. Amplitude E_3 for $k_3 = 1.5$

V. CONCLUSION

The explicit solution of the EM wave scattering problem for thin cylinder is derived using the asymptotical approach. The main parameters, which influence on the characteristic of solution, are the both radius of cylinder a and parameter k_3 describing the change of the medium properties along the z axis.

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