Modeling of Nonlinear Fish-Scale Metamaterial via Coupled Duffing Oscillators

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Abstract - T**he dynamic system of two coupled Duffing oscillators is considered in order to predict the optical response of the nonlinear planar fish-scale metamaterial. The direct numerical calculation of metamaterial response confirms the correctness of the proposed model.**

Keywords – **Duffing oscillator, nonlinear metamaterial.**

I. INTRODUCTION

The metamaterials are wide class of artificial media which have special electromagnetic properties. Using such materials enable to create different frequency filters, transformers of wave polarization and other devices. In order to broaden ability of linear (traditional) metamaterials the nonlinearity is included in their composition [1]. Nevertheless, the considerations of nonlinear metamaterials demand to overcome some theory difficulties. The development of reductive methods for studying optical properties of nonlinear metamaterials is an important problem.

In this report we propose a model of coupled Duffing oscillators to predict the optical response of nonlinear planar bilayered fish-scale metamaterial [2].

II. PROBLEM STATEMENT AND SOLUTION

We consider a planar metamaterial which consists of two equidistant gratings of continuous meandering strips placed on the both sides of a thin dielectric substrate (bilayered fish-scale structure, Fig. 1). The structure under study possesses resonant properties which particularly appear in result of the trapped-mode excitation [3, 4].

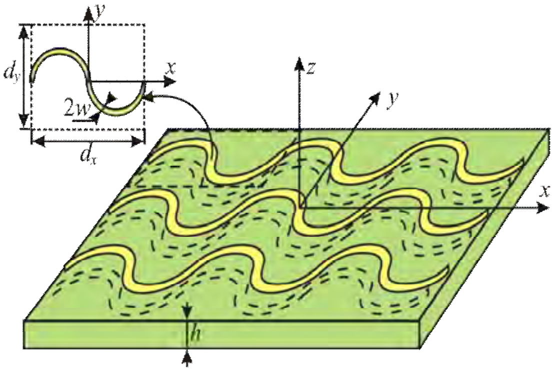

Fig.1 Fragment of planar fish-scale metamaterial.

Due to the bilayered configuration of the structure under study there are two possible current distributions which correspond to the trapped–mode resonances. The first distribution is the antiphased current oscillations in the arcs of each grating. The second distribution is the antiphased current oscillations between two adjacent gratings. Thus generally the structure consists of two systems which provide the current oscillations and the coupling between them is realized via the dielectric substrate of the metamaterial. The main interest is to find out what changes appear in the optical response of the metamaterial when the nonlinearity is introduced into the system. In this report we consider the metamaterial which substrate is made of Kerr-type nonlinear dielectric when its permittivity depends on the field intensity inside the structure.

In order to predict the approximate behaviour of the metamaterial with nonlinear substrate we construct a model based on the theory of oscillations. The interaction of two currents through nonlinear dielectric we describe via two coupled Duffing oscillators with harmonic driving force applied to one of them:

$$\begin{cases} \dfrac{d^2x_1}{dt^2}+g_1\dfrac{dx_1}{dt}+w_1^2x_1+b_1x_1^3+c_1x_1x_2=A\exp(iwt), \\ \dfrac{d^2x_2}{dt^2}+g_2\dfrac{dx_2}{dt}+w_2^2x_2+b_2x_2^3+c_2x_1x_2=0, \end{cases} \tag{1}$$

where g_j are the damping ratios, w_j are the resonant frequencies, b_j are the nonlinearity parameters and c_j are the coupling coefficients ($j=1,2$).

In the linear case ($b_1=b_2=0$) the steady-state harmonic solutions are: $x_1(t)=C_1(w)\exp(iwt)$, $x_2(t)=C_2(w)\exp(iwt)$ [5], with the frequency dependent amplitudes $C_1(w)$ and $C_2(w)$ obtained in the form:

$$\begin{aligned} C_1(w)&=\frac{w_2^2-w^2+ig_2w}{(w_1^2-w^2+ig_1w)(w_2^2-w^2+ig_2w)-c_1c_2}A, \\ C_2(w)&=-\frac{c_2}{(w_1^2-w^2+ig_1w)(w_2^2-w^2+ig_2w)-c_1c_2}A. \end{aligned} \tag{2}$$

The amplitude of the first oscillator as a function of the frequency of an external force is shown in Fig. 2 with the solid line. Two resonant peaks appear in the chosen frequency window, and their location corresponds to the real parts of the complex eigen-frequencies of the system (1). It can be seen that one of the resonances has asymmetric form with zero condition on the right side near the peak position. This condition appears at the frequency $w=w_2$ when $g_2=0$. Note that such asymmetrical form of resonance is actively discussed in relation to the theories of Fano-shape resonances and electromagnetically-induced transparency [4, 5].

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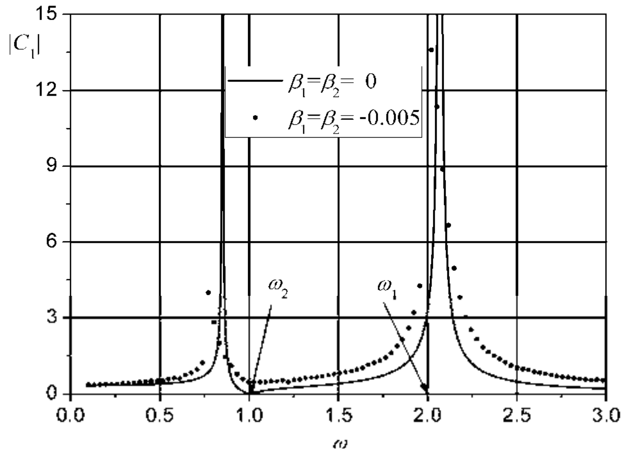

Fig. 2. The resonant behaviour of the amplitude of the first oscillator in the coupled system: $A=1$, $w_1=2$, $w_2=1$, $c_1=c_2=0.95$, $g_1=g_2=0.001$.

Next consider the nonlinear case when $b_j \neq 0$. Unfortunately the solution of nonlinear system (1) cannot be written in a simple analytical form like (2). In order to find the similar amplitude expressions in the nonlinear case we use a numerical method. At the first stage, for each certain frequency, the Cauchy problem with zero initial conditions related to the system (1) in the case of the real-valued driving force ($A\cos(wt)$) is solved using the forth-order Runge-Kutta method. At the second stage, the maximal value of the obtained solution for each frequency is found. The frequency dependence of the maximal values of solutions is presented in Fig. 2 with dot line.

In the nonlinear regime the resonant curves have typical form of the bended resonances and the ambiguity (bistability) appears. The direction of this bending depends on the sign of the parameter b. The bending and ambiguity produce the effect of the amplitude hysteresis of forced oscillations under slow variation of the driving force frequency. It manifests itself in the stepwise changing of the amplitude with frequency increasing or decreasing. Evidently that in the system of two coupled oscillators under certain conditions the overlapping of the resonant curves can appear which brings the system in the multistable state.

This circumstance is depicted in Fig. 3 where typical curves of the averaged current magnitude and the transmission coefficient magnitude of the nonlinear metamaterial are given as functions of the frequency. To obtain these curves, the nonlinear problem of wave diffraction on the planar metamaterial is solved numerically. One can see that in the linear regime the calculated curves have similar form (see the solid lines in the Fig. 2 and Fig. 3a). Also when the nonlinearity increases, the frequency dependences of the averaged current magnitude takes a form of the bent resonances.

An important point is that this bending is different for the first and second resonances and under certain conditions the second resonance overlaps the first one which brings the system in the multistable state. From the Fig. 3b one can see that such dependence of the current magnitude forms the complicate feature of the transmission spectra of the metamaterial.

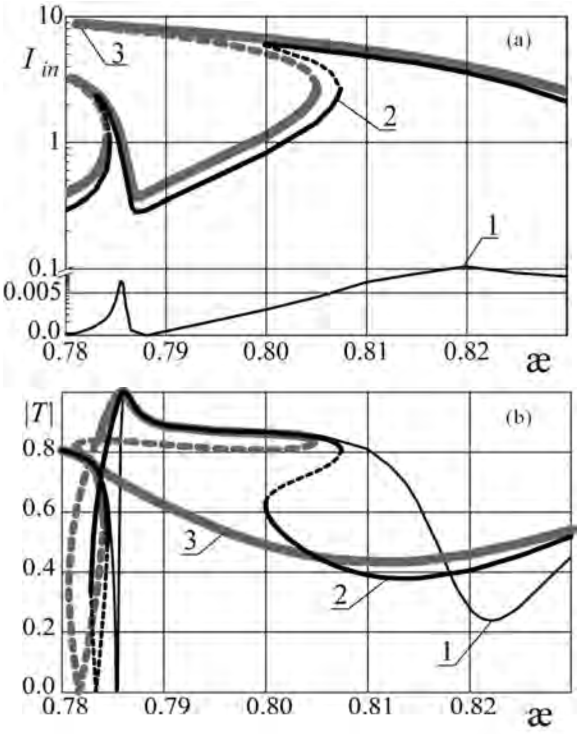

Fig. 3. The frequency dependences of the averaged current magnitude (a) and the transmission coefficient (b) magnitudes versus the different incident field intensity in the case when permittivity of the metamaterial substrate depends on the field intensity. Here I_0 is the input field intensity, æ is the dimensionless frequency. Curve 1 - $I_0 = 1$ kW/cm^2, curve 2 - $I_0 = 200$ kW/cm^2, curve 3 - $I_0 = 300$ kW/cm^2.

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