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THE RECOGNITION OF DISCRETE PATTERNS AND SIGNALS IN THE NEURAL BASIS

The method of real time synthesis of four-layered neural schemes for the recognition of Boolean vectors is studied in the paper. Also studied is the efficiency of the working of these schemes dependent upon the values of the tolerance matrix indices used in the recognition schemes synthesis.

Key Words: Tolerance matrix, *p*-cover, threshold element, neural basis, learning sample.

Запропоновано метод синтезу чотирьохшарової нейромережі для розпізнавання об'єктів, закодованих бульовими векторами. В основі цього метода є *p*-розклад множин бульових векторів і синтез нейроелементів методом матриць толерантності.

Ключові слова: матриця толерантності, *p*-покриття, нейроелемент, нейробазис, навчальна вибірка.

Introduction

The methods of the neural technologies are very powerful and useful for solving different applied tasks in information theory, time series forecasting and pattern recognition. The synthesis procedure consists of two main levels. In the first level we must elaborate efficient methods of the synthesis of one threshold device with many inputs. The number of inputs depends on actual task conditions. The second level is connecting the neural elements into one logical schema. The configuration of this schema must provide the realization of needed mapping.

The classical threshold synthesis methods (approximation methods, iterative methods) aren't efficient in practice if the neuron have large numbers of inputs. So the development of new methods of synthesis of threshold elements of neural schemas is a very important task nowadays.

In the given investigation we proposed the method of tolerance matrices for the synthesis of one threshold element. We also proposed the Boolean vectors *p*-cover method for the synthesis of the logical schemas of threshold devices.

The given methods effectively resolve the problems of synthesis of the four-layered nets with the large number of inputs. These nets can be successfully used in solving the problem of real task classification and object recognition encoded by Boolean vectors.

The synthesis of the recognition scheme

Let $K_1, K_2, \mathbf{K}, K_t$ be the learning sample for object classes $K'_1, K'_2, \mathbf{K}, K'_t$ | Classes $K'_i,\ K'_j\ (i \neq j)$ and subsets $K_i \subset K'_i,\ K_j \subset K'_j\ (i \neq j)$ can have non-empty intersection.

Let us consider the task of the neural scheme synthesis assigning for several objects d from $\bigcup_{i=1}^{t} K'_i$ one of the object classes K'_i if the learning sample is thus defined:

$$K_1 = \left\{ \left(a_{11}^{(1)}, \mathbf{K}, a_{1n}^{(1)}\right), \left(a_{21}^{(1)}, \mathbf{K}, a_{2n}^{(1)}\right), \mathbf{K}, \left(a_{k_1 1}^{(1)}, \mathbf{K}, a_{k_1 n}^{(1)}\right) \right\},$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$K_t = \left\{ \left(a_{11}^{(t)}, \mathbf{K}, a_{1n}^{(t)}\right), \left(a_{21}^{(t)}, \mathbf{K}, a_{2n}^{(t)}\right), \mathbf{K}, \left(a_{k_1 t}^{(t)}, \mathbf{K}, a_{k_1 n}^{(t)}\right) \right\},$$

where $a_{ij}^{(r)} \in \{0,1\}$.

We use the neural scheme with three neural device layers and one logical output block which detects the membership of the specified object d to one of the object classes $K'_1, K'_2, \mathbf{K}, K'_t$.

If each of the third-layer output functions' values is less than $\eta\ h \in (0;1)$, where η is the recognition threshold and its value is defined by a learning sample (the minimal value $h = h^*$ ensure the minimal learning set error) then the recognition scheme does not make a decision. Our recognition scheme can be depicted in the following manner:

According to Figure 1, every class K_i corresponds to the block with the output value F_i. We need to find first-layer weight vectors for the recognition scheme synthesis

$$\mathbf{w}_{11} = \left(w_1^{11}, \mathbf{K}, w_n^{11}; T_1^1\right), \mathbf{K}, \mathbf{w}_{r_1 1} = \left(w_1^{r_1 1}, \mathbf{K}, w_n^{r_1 1}; T_{r_1}^1\right),$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\mathbf{w}_{1t} = \left(w_1^{1t}, \mathbf{K}, w_n^{1t}; T_1^t\right), \mathbf{K}, \mathbf{w}_{r_t t} = \left(w_1^{r_1 t}, \mathbf{K}, w_n^{r_1 t}; T_{r_1}^t\right)$$

and second-layer weight vectors $\mathbf{v}_1 = \left(v_1^1, \mathbf{K}, v_{r_1}^1\right), \mathbf{K}, \mathbf{v}_t = \left(v_1^t, \mathbf{K}, v_{r_t}^t\right)$. The quantities f_r^k, g_1^k, g_2^k and $F_k\ (k = 1, 2, \mathbf{K}, t)$ are defined thus:

$$f_r^k(x_1, \mathbf{K}, x_n) = \begin{cases} 1, & \text{if } w_1^{r_k} x_1 + \mathbf{K} + w_n^{r_k} x_n \geq T_r^k, \\ 0, & \text{if } w_1^{r_k} x_1 + \mathbf{K} + w_n^{r_k} x_n < T_r^k, \end{cases}$$

$r \in \{1, 2, \mathbf{K}, r_k\}$,

$$g_1^k = v_1^k f_1^k + \mathbf{K} + v_r^k f_r^k, \quad g_2^k = f_1^k + \mathbf{K} + f_r^k + 1$$

and $F_k = \dfrac{g_1^k}{g_2^k}$.

Let us consider the method of synthesis of the first- and second-layer neurons $\mathbf{w}_{11}, \mathbf{K}, \mathbf{w}_{r_1 1}, \mathbf{K}, \mathbf{w}_{1t}, \mathbf{K}, \mathbf{w}_{r_t t}, \mathbf{v}_1, \mathbf{K}, \mathbf{v}_t$.

Let A – be the arbitrary subset of the set Z_2^n, $\mathbf{a} \in A$ and p be the threshold operator [1] with the label σ and the index j. Then $p(\mathbf{a}A)$ is the maximum subset of $\mathbf{a}A$, that

$$p(\mathbf{a}A)_x^s = \left(L_j 0_j\ \mathbf{K}\, 0_j\right) \Delta \left(\mathop{\Delta}_{i=0}^{n-j} \left(L_{j+i}^*(q_i)\, 0\, \mathbf{K}\, 0_{n-(j+i)} \right) \right), \qquad (1)$$

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where $q_0 \geq q_1 \mathbf{K} \geq q_{n-j}$.

The index j in (1) is also called p-subset $\{\mathbf{a}\, p(\mathbf{a}A)\}$ index.

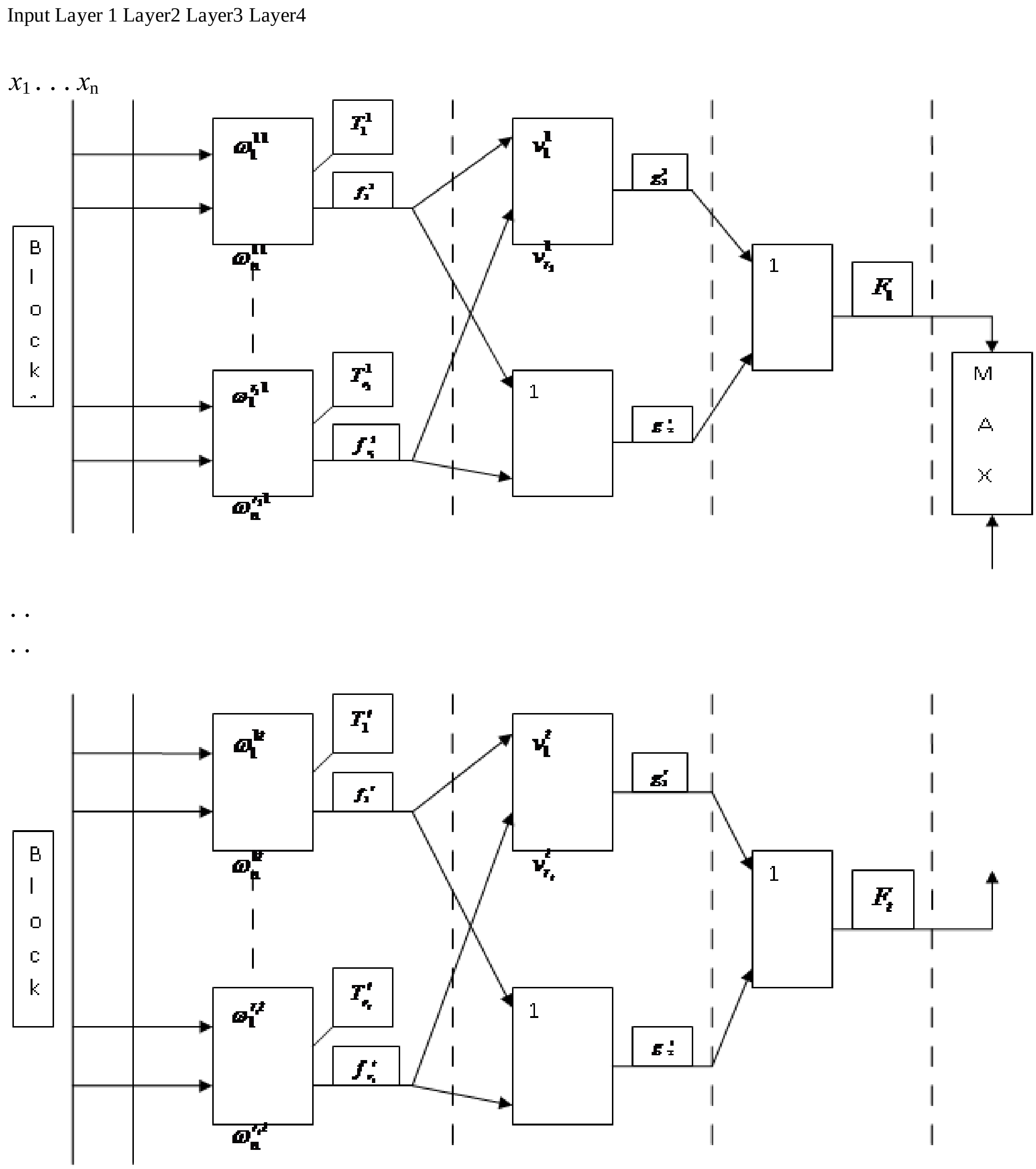

Figure 1. Recognition scheme in neural basis

Note: The label $s \in S_n$ is defined so that the units' quantity $K(i)$, $K(i+1)$ of the columns i, $i+1$ satisfies inequality $K(i) \geq K(i+1)$.

An arbitrary Boolean vector set A_s can be thus described:

$$A_s = \mathbf{a}_1^s p(\mathbf{a}_1^s A_s) \cup \mathbf{a}_2^s p(\mathbf{a}_2^s A_s) \cup \mathbf{K} \cup \mathbf{a}_{r_s}^s p(\mathbf{a}_{r_s}^s A_s),$$

where:

$$\mathbf{a}_i^s p(\mathbf{a}_i^s A_s) \not\subset \bigcup_{j=1}^{i-1} \mathbf{a}_j^s p(\mathbf{a}_j^s A_s), \tag{2}$$

The points $\mathbf{a}_1^s, \mathbf{K}, \mathbf{a}_{r_s}^s$ are called decomposition points of the set A_s on p-subset with the indices $j_{1s}, \mathbf{K}\, j_{r_s s}$. The decomposition points $\mathbf{a}_1^s, \mathbf{K}, \mathbf{a}_{r_s}^s$ of the set A_s on p-subsets are selected so that the count of the unitary vectors of the set $\mathbf{a}_i^s A_s$ is no smaller than the count of the unitary vectors of the set $\mathbf{a}_{i+1}^s A_s$ $(i=1,2,\ldots r_s-1)$ and that the condition (2) is satisfied. Later we will consider only such decomposition points.

Let B_s be the arbitrary subset of the set $A_s \subset Z_2^n$ and $\mathbf{b}_1^s, \mathbf{b}_2^s, \ldots, \mathbf{b}_{r_s}^s$ be such vectors from B_s that

$$B_s \subseteq \mathbf{b}_1^s p\left(\mathbf{b}_1^s A_s\right) \cup \mathbf{b}_2^s p\left(\mathbf{b}_2^s A_s\right) \cup \ldots \cup \mathbf{b}_{r_s}^s p\left(\mathbf{b}_{r_s}^s A_s\right) \tag{3}$$

and

$$B_s \not\subset \bigcup_{i=1, i\neq j}^{r_s} \mathbf{b}_i^s p\left(\mathbf{b}_i^s A_s\right), \tag{4}$$

where $j \in \{1, \ldots, r_s\}$ and *p*-subsets $\mathbf{b}_i^s p\left(\mathbf{b}_i^s A_s\right)$ satisfy (2).

The system of the *p*-subsets $\mathbf{b}_1^s p\left(\mathbf{b}_1^s A_s\right), \ldots, \mathbf{b}_{r_s}^s p\left(\mathbf{b}_{r_s}^s A_s\right)$ define the *p*-cover of the subset B_s in the set A_s relative to points $\mathbf{b}_1^s, \ldots, \mathbf{b}_{r_s}^s \in B_s$ with the corresponding indices $j_{1s}, j_{2s}, \ldots, j_{r_s s}$, if they satisfy condition (2)-(4). If we denote the set $\bigcup_{i=1}^{r_s} \mathbf{b}_i^s p\left(\mathbf{b}_i^s A_s\right)$ as $P_{A_s}\left(B_s; \mathbf{b}_1^s, \ldots, \mathbf{b}_{r_s}^s\right)$ then it is evident that $B_s \subseteq P_{A_s}\left(B_s; \mathbf{b}_1^s, \ldots, \mathbf{b}_{r_s}^s\right) \subseteq A_s$.

We called the minimum *p*-cover of the set B_s in A_s relative to points $\mathbf{b}_1^s, \ldots, \mathbf{b}_{r_s}^s \in B_s$ with the corresponding indices $j_{1s}, j_{2s}, \ldots, j_{r_s s}$ of its components $\mathbf{b}_i^s p\left(\mathbf{b}_i^s A_s\right)$ $(i=1,\ldots,r_s)$ such a minimum subset $P_{A_s}^{\min}\left(B_s; \mathbf{b}_1^s, \ldots, \mathbf{b}_{r_s}^s\right)$ in A_s, that (2)-(4) are satisfied.

The maximum *p*-cover $P_{A_s}^{\max}\left(B_s; \mathbf{b}_1^s, \ldots, \mathbf{b}_{r_s}^s\right)$ of the set B_s in set A_s is similarly defined.

It is evident that if $P_{A_s}^{\min}\left(B_s; \mathbf{b}_1^s, \ldots, \mathbf{b}_{r_s}^s\right) \neq P_{A_s}^{\max}\left(B_s; \mathbf{b}_1^s, \ldots, \mathbf{b}_{r_s}^s\right)$ then exists *p*-cover $P_{A_s}\left(B_s; \mathbf{b}_1^s, \ldots, \mathbf{b}_{r_s}^s\right)$ so that $P_{A_s}^{\min}\left(B_s; \mathbf{b}_1^s, \ldots, \mathbf{b}_{r_s}^s\right) \subset P_{A_s}\left(B_s; \mathbf{b}_1^s, \ldots, \mathbf{b}_{r_s}^s\right) \subset P_{A_s}^{\max}\left(B_s; \mathbf{b}_1^s, \ldots, \mathbf{b}_{r_s}^s\right)$.

Let the *p*-subset $p\left(\mathbf{b}_m^s A_s\right)$ of the set $\mathbf{b}_m A_s$ $(m \in \{1, \ldots, r_s\})$ have the label s_{ms} and the index. Then

$$p\left(\mathbf{b}_m^s A_s\right)_{x_{ms}}^{s_{ms}} = \left(L_{j_{ms}}\ 0_{j_{ms}}\ \ldots 0_{j_{ms}}\right) \Delta \left(\mathop{\Delta}_{i=0}^{n-j_m} \left(L_{j_{ms}+i}^{*}\left(q_i^{ms}\right)\ \underbrace{0 \ldots 0}_{n-(j_{ms}+i)}\right)\right), \tag{5}$$

where $q_0^{ms} \geq q_1^{ms} \ldots \geq q_{n-j_{ms}}^{ms}$.

Let k_{ms} be the smaller entire non-negative number that $q_{k_{ms}}^{ms} \neq 0$ and $q_{k_{ms}+1}^{ms} = 0$. Let us build the *n*-dimensional vector $\mathbf{u}_{ms} = \left(u_1^{ms}, \ldots, u_{j_{ms}}^{ms}, u_{j_{ms}+1}^{ms}, \ldots, u_n^{ms}\right)$ thus:

$$u_1^{ms} = -1,\ u_2^{ms} = u_1^{ms} - 1, \ldots, u_{j_{ms}}^{ms} = \sum_{i=1}^{j_{ms}-1} u_i^{ms} - 1,$$

$$u_{j_{ms}+1}^{ms} = u_{j_{ms}}^{ms} + \left(q_1^{ms} - q_0^{ms}\right), \ldots,\ u_{j_{ms}+k_{ms}}^{ms} = u_{j_{ms}+k_{ms}-1}^{ms} + \left(q_{k_{ms}}^{ms} - q_{k_{ms}-1}^{ms}\right),$$

$$u_{j_{ms}+k_{ms}+1} = u_{j_{ms}+k_{ms}+2} = \ldots = u_n^{ms} = \left(g_{k_{ms}}^{ms}, c_{k_{ms}}^{ms}\right) - 1,$$

where $g_{k_{ms}}^{ms}$ – is the last row of the matrix $\left(L_{j_{ms}+k_{ms}}^{*}\left(q_{k_{ms}}\right)\ 0 \ldots 0\right)$, $c_{k_{ms}}^{ms} = \left(u_1^{ms}, \ldots, u_{j_{ms}+k_{ms}}^{ms}\ 0, \ldots, 0\right)$ – is the *n*-dimensional vector and $\left(g_{k_{ms}}^{ms}, c_{k_{ms}}^{ms}\right)$ – is the scalar product of $g_{k_{ms}}^{ms}$ and $c_{k_{ms}}^{ms}$. It is simple to notice that $\mathbf{u}_{ms}$ satisfies the following condition

$$\forall \mathbf{x} \in p\left(\mathbf{b}_m^s A_s\right) \text{ i } \forall \mathbf{y} \in Z_2^n \setminus p\left(\mathbf{b}_m^s A_s\right)\ (\mathbf{x}, \mathbf{u}_{ms}) > (\mathbf{y}, \mathbf{u}_{ms}).$$

Let $\mathbf{g} = (g_1,...,g_n)$ be the last row of the matrix $p(\mathbf{b}^s_m A_s)^{s_{ms}}_{x_{ms}}$. Let the vector $\mathbf{w}_{ms} = \mathbf{b}^s_m(\mathbf{u}^{s^{-1}_{ms}}_{ms})$ and the number $T^s_m = ((\mathbf{b}^s_m \oplus \mathbf{g}^{s^{-1}_{ms}}), \mathbf{w}_{ms})$, where $\oplus$ is the addition mod 2. If $\mathbf{w}_{ms}$ is the weight vector of the neuron and its threshold is equal to T^s_m then the output signal of the neuron is equal to 1 only when the input vector belongs to $\mathbf{b}^s_m p(\mathbf{b}^s_m A_s)$. So, the characteristic function f^s_m of the set $\mathbf{b}^s_m p(\mathbf{b}^s_m A_s)$ is a neural function realizable with the single neural element with the structure vector $[\mathbf{w}_{ms}; T^s_m]$. If s is fixed and m run the set $\{1,...,r_s\}$ we can synthesize in such manner all neurons of the block number s of the first layer, where $s \in \{1,...,t\}$. Unlike to the synthesis algorithm of the recognition scheme in [3], the weight vectors $\mathbf{v}_1 = (v^1_1,...,v^1_{r_1}),...,\mathbf{v}_t = (v^t_1,...,v^t_{r_t})$ of the second layer can be thus calculated:

$$v^s_m = \frac{\left|\mathbf{b}^s_m p(\mathbf{b}^s_m A_s) \cap B_s\right|}{|B_s|}, \tag{6}$$

where $m \in \{1,...,r_s\}$, $s \in \{1,..t\}$ and $|A|$ – is the power of the set A.

If the learning sample is not specified, then $\max F_i$ is chosen as decision without comparing it to the learning threshold η if $\max F_i \neq 0$, and we do not make decision if $\max F_i = 0$.

If the learning sample is specified, then the synthesis of the recognition scheme is connected with the learning using the value of η (in this case $\max F_i$ is compared with η and a decision is made only when $\max F_i \geq h$). Then we build the optimal recognition scheme [4] for the present learning sample.

The algorithm of the synthesis of the recognition scheme

Step 1. Let $\{K_1,...,K_t\}$ be the learning sample, set $s = 1$ and go to step 2.

Step 2. Build the set

$$A_s = K_s \cup \left(Z^n_2 \setminus \bigcup_{i \neq s} K_i \right) \tag{7}$$

and find an arbitrary p-cover of the set K_s with the fixed indices $j_{1s}, j_{2s},..., j_{r_s s}$ in the set A_s, that is

$$P_{A_s}(K_s; \mathbf{b}^s_1,...,\mathbf{b}^s_{r_s}) = \mathbf{b}^s_1 p(\mathbf{b}^s_1 A_s) \cup \mathbf{b}^s_2 p(\mathbf{b}^s_2 A_s) \cup \mathbf{K} \cup \mathbf{b}^s_{r_s} p(\mathbf{b}^s_{r_s} A_s).$$

Search out the structure vector $[\mathbf{w}_{ms}; T^s_m]$ for the characteristic function f^s_m of the set $\mathbf{b}^s_m p(\mathbf{b}^s_m A_s)$ $(m = 1,2,...,r_s)$ following the above-mentioned algorithm. On detecting the first-layer structures $\{[\mathbf{w}_{1s}; T^s_1],...,[\mathbf{w}_{r_s s}; T^s_{r_s}]\}$, search out the second-layer weight vector $\mathbf{v}_s$ and go to step 3.

Step 3. If $s < t$ then set $s = s + 1$ and go to step 2, otherwise the synthesis of the scheme is finished.

Note 1. *P*-cover $p_{A_s}(K_s; \mathbf{b}^s_1,....,\mathbf{b}^s_{r_s})$ of the set K_s in A_s with the fixed indices $j_{1s},..., j_{sr_s}$ is not detected unambiguously. Therefore we can select minimum, maximum or any other p-cover. If we want to select the p-cover unambiguously, then we should define the learning sample (the set of Boolean vectors for the testing of the recognition scheme) and select for it the optimal p-cover of the set K_s in A_s with the fixed indices $j_{1s},..., j_{sr_s}$ with the property of minimizing the error for this learning sample for fixed η. The step-by-step changing of the value of $h \in (0, 1)$ helps us to search out the optimal value h^* and respective optimal indices

$j^*_{1s},..., j^*_{r_s s}$ of the p-subsets.

Note 2. Subject to the technical condition of first-layer neuron realization (the maximum absolute value of weights) we can detect the index j_{ms} of the tolerance matrix in (5) corresponding to the given calculation. Following the above-mentioned algorithm the maximum absolute value of the weights of the block s is equal to $2^{j_{ms}-1} + 1$. Therefore the bounds at components of structure vectors are defined by the indices of the tolerance matrix.

Note 3. If we set $j_{1s}=1,\ldots,j_{sr_s}=1$ we should obtain the simple classifier.

Let us consider the example of the synthesis scheme. Let us have the receptive fields 3×3. Set the following learning sample and specify two classes

class 1.

class 2.

We thus encode these binary images. The first three components of vector are filled on the basis of the first line of the receptive field. If the corresponding cell contains "*", then the component of vector is equal to 1. If the cell does not contain "*", then it is equal to 0. Next we fill following components on the base of second and third rows.

Let us write the codes of respective binary images in the learning sample classes $K_1;\ K_2$ of the classes $K_1',\ K_2'$

$$K_1=\begin{cases}\mathbf{b}_1^1=(010111010),\\ \mathbf{b}_2^1=(010011010),\\ \mathbf{b}_3^1=(010101010),\end{cases}\qquad K_2=\begin{cases}\mathbf{b}_1^2=(111010010),\\ \mathbf{b}_2^2=(110010010),\\ \mathbf{b}_3^2=(011010010).\end{cases}$$

We find the maximum p-cover for $K_1',\ K_2'$ with the maximum indices and synthesize the recognition scheme. According to the algorithm, on building the p-set $p\left(\mathbf{b}_1^1A_1\right)^{s_{11}}$,where $s_{12}=\begin{pmatrix}1&2&3&4&5&6&7&8&9\\3&4&5&1&2&6&7&8&9\end{pmatrix}$, we can use all vectors from Z_2^9 except $(101011000)^{s_{11}}$, $(101001000)^{s_{11}}$, $(100011000)^{s_{11}}$. So

$$p\left(\mathbf{b}_1^1A_1\right)^{s_{11}}=\left(L_2\ \ 0\ldots0\right)\Delta\left(L_2^*(1)\ \ 0\ldots0\right)\Delta\left(L_3^*(1)\ \ 0\ldots0\right)\Delta\ldots\Delta\left(L_9^*(1)\ \ 0\ldots0\right).\tag{8}$$

We can write (8) at matrix notation

$$p\left(\mathbf{b}_1^1A_1\right)^{s_{11}}=\begin{pmatrix}000000000\\100000000\\010000000\\001000000\\000100000\\000010000\\000001000\\000000100\\000000010\\000000001\end{pmatrix}$$

The maximum p-cover of the learning sample K_1 of the class K_1' in A_1 contains the only subset, that is

$$P_{A_1}\left(K_1;\mathbf{b}_1^1\right)=\mathbf{b}_1^1 p\left(\mathbf{b}_1^1 A_1\right).$$

On the basis of decomposition (8) we build the vector $\mathbf{u}_{11}=\left(u_1^{11},u_2^{11},...,u_9^{11}\right)$:

$$u_1^{11}=-1,\ u_2^{11}=u_1^{11}-1=-2,\ u_3^{11}=u_2^{11}+(1-1)=-2,...,u_9^{11}=u_8^{11}+(1-1)=-2.$$

Then we define the weight vector $\mathbf{w}_{11}=\mathbf{b}_1^1\left(\mathbf{u}_{11}^{s_{11}^{-1}}\right)=\mathbf{b}_1^1(-2,-2,-2,-1,-2,-2,-2,-2)=(-2,2,-2,1,2,2,-2,2,-2)$ and the threshold $T_1^1=\left(\left(\mathbf{b}_1^1\oplus\mathbf{g}^{s_{11}^{-1}}\right),\mathbf{w}_{11}\right)=\left((0,1,0,1,1,1,0,1,1),\mathbf{w}_{11}\right)=7$. So, we look for the first-layer neuron of the first block with the structure [(–2,–2,–2,1,2,2,–2,2,–2);7].

Similarly we can build the vector structure of the neural element of the second block of the first layer.

$$p\left(\mathbf{b}_1^2 A_2\right)^{s_{12}}=\begin{pmatrix}000000000\\100000000\\010000000\\001000000\\000100000\\000010000\\000001000\\000000100\\000000010\\000000001\end{pmatrix},$$

where $s_{12}=\begin{pmatrix}1&2&3&4&5&6&7&8&9\\1&3&2&4&5&6&7&8&9\end{pmatrix}$. Here we can use all vectors from Z_2^9 except $(101011000)^{s_{12}}$, $(101001000)^{s_{12}}$, $(100011000)^{s_{12}}$. In this case the maximum p-cover of the learning sample from the class K_2' in A_2 with the maximum index coincides with the set $\mathbf{b}_1^2 p\left(\mathbf{b}_1^2 A_2\right)$, that is

$$P_{A_2}\left(K_2;\mathbf{b}_1^2\right)=\mathbf{b}_1^2 p\left(\mathbf{b}_1^2 A_2\right).$$

The above-mentioned algorithm $\mathbf{u}_{12}=(-1,-2,-2,-2,-2,-2,-2,-2,-2)$,

$\mathbf{w}_{12}=\mathbf{b}_1^2\left(\mathbf{u}_{12}^{s_{12}^{-1}}\right)=(1,2,2,-2,2,-2,-2,2,-2)$ and $T_1^2=\left(\left(\mathbf{b}_1^2\oplus\mathbf{g}^{s_{12}^{-1}}\right),\mathbf{w}_{12}\right)=7$. Then, the neuron of the second block has the structure $\left[\mathbf{w}_{12},T_1^2\right]$.

Then we substitute K_s in place of B_s in (6) and find $v_1^1=1,\ v_1^2=1$. Thereby, the synthesis of the recognition scheme is finished. The corresponding network has the following view:

The first-layer neuron of block 1 can be activated only if input vectors belong to

$$P_{A_2}\left(K_1;\mathbf{b}_1^1\right)=\mathbf{b}_1^1 p\left(\mathbf{b}_1^1 A_1\right)=\{(010111010),(010011010),(010101010),(110111010),$$
$$(000111010),(011111010),(010110010),(010111110),(010111000),(010111011)\},$$

and these vectors cannot activate the first-layer neuron of block 2. Therefore, the recognition scheme reckons the following images to class K_1'

These images correspond to the set $P_{A_2}\left(K_2;\mathbf{b}_1^2\right)=\mathbf{b}_1^2 p\left(\mathbf{b}_1^2 A_2\right)=\{(111010010),(011010010),(110010010),$
$(101010010),(111110010),(111000010),(111011010),(111010110),(111010000),(111010011)\}$.

Concerning the others binary images the recognition scheme doesn't make a decision because $F_1=0$ and $F_2=0$.

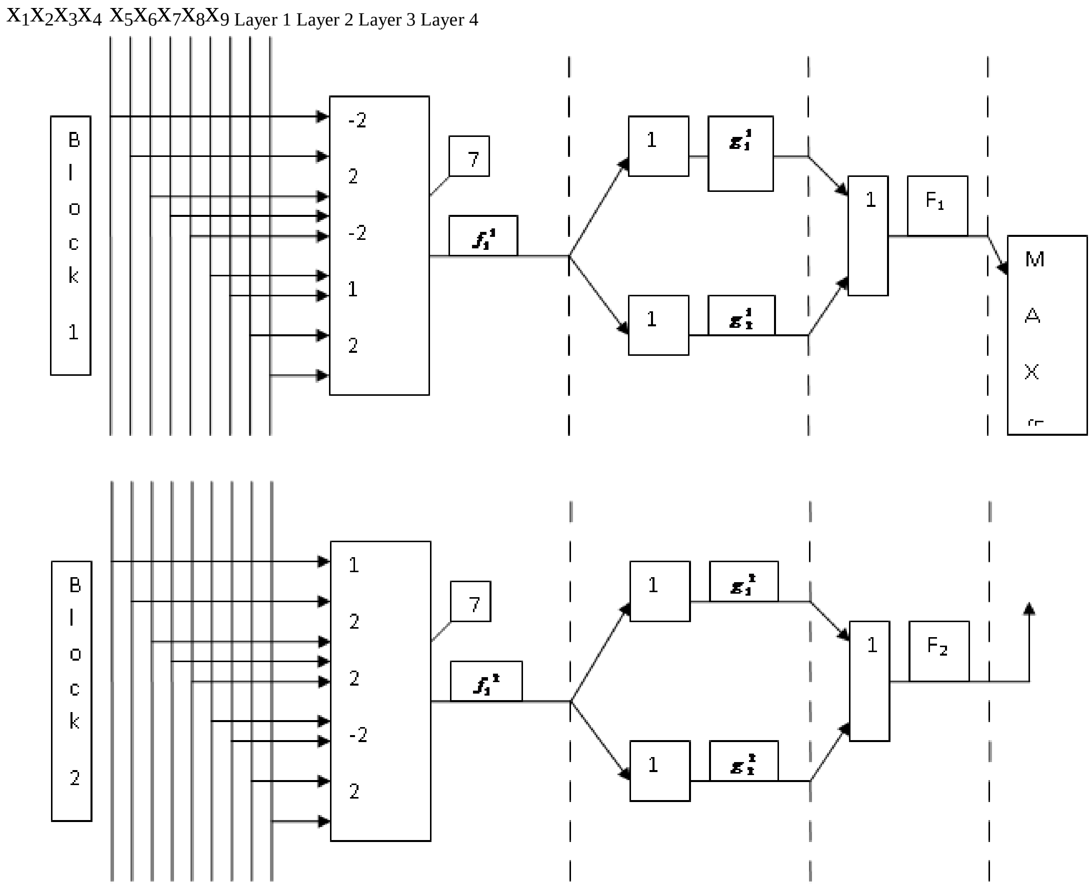

Figure 2. Recognition scheme of image

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According to the recognition scheme the following images are included to the class K_2'

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If we realize the synthesis of the recognition scheme (fig. 2) for above-mentioned classes of binary images concerning minimum *p*-covers with the minimum indices then we should have

$$p\left(\mathbf{b}_1^1 A_1\right)^{s_{11}} = p\left(\mathbf{b}_1^2 A_2\right)^{s_{12}} = \begin{pmatrix} 000000000 \\ 100000000 \\ 010000000 \end{pmatrix}$$

and $P_{A_1}\left(K_1;\mathbf{b}_1^1\right) = K_1$, $P_{A_2}\left(K_2;\mathbf{b}_1^2\right) = K_2$. The first-layer neurons would have the following structures:

(Block 1): [(–1,1,–1,1,1,1,–1,1–1);4], (Block 2): [(1,1, 1,–1,1,–1,–1,1–1);4] with the recognition scheme being the simple classifier.

Note 1. If the probabilities p_i of the occurrence of the vectors of classes $K_1',...,K_t'$ are not equal then the recognition scheme (fig. 1) is thus modified: in the fourth layer $\max\{F_i\}$ is changed by $\max\{p_i F_i\},\ i = 1,...,t$.

Conclusion

1. The developed method of the synthesis of four-layered neural network works in real time and can be useful for the recognition of the object classes encoded by Boolean vectors of very large length.

2. This method provides the searching procedure for optimal values of parameters of the recognition schema that can improve its precision.

3. The method of the synthesis includes different constrains on the value of the first layer weight vectors. It is very important for the technical realizations.

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