

INVESTIGATION OF INTERMITTENT CHAOTIC PROCESSES IN
PIECEWISE CONTINUOUS SYSTEMS

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Abstract. The paper discloses the structure of chaotic characteristic of the piecewise continuous systems, and it is shown that the observed processes represent a new class of motions of physical objects, consisting of alternating chaotic oscillations, each of which consists of an integral, nonperiodic, fractal and deterministic sequence of chaotic component to its inherent ongoing process.

Keywords: chaos, PWM, mapping, fractal.

1. Introduction

The study of deterministic chaotic processes in piecewise continuous systems [1,2,3,4] was formed as an independent scientific direction [5], where the existence of deterministic chaos in these systems is confirmed, the evaluation of correlation functions and the location of the poles of the characteristic equation is performed [6, 7]. The absence of such a parameter as a constant process (for example, the Feigenbaum constant) indirectly indicates the features of these processes [8].

2. Model description

Let's consider a system described by a piecewise-continuous differential and the functional equation:

$$\begin{cases} \frac{dX(t)}{dt} = A(t, \gamma_1, \dots, \gamma_n) X(t) + B(t, \gamma_1, \dots, \gamma_n), & (1) \\ F(X(t), Y(t)) = 0; \end{cases}$$

where X(t) – the vector of independent variables; Y(t) – a periodic function; A(t, $\gamma_1, \dots, \gamma_n$) and B(t, $\gamma_1, \dots, \gamma_n$) – respectively the coefficient matrix and vector effects, depending on the variables $\gamma_1, \dots, \gamma_n$ ($n = 1, 2, 3, \dots$), which take the value either - "0" or "1" depending on the value of the function F(X(t), Y(t)). The number of variables $\gamma_1, \dots, \gamma_n$ corresponds to the number of roots of the function F(X(t), Y(t)), while addressing specific variables γ_n in the system of equations (1) to the law of the system.

In the system described by equations (1), apart from the known ones, the possible process consists of alternating integer fractal sequences corresponding to the inherent deterministic chaotic oscillations.

Let's demonstrate this assertion by the following example. Let's consider the electric circuit (Fig.1), used as a down DC converter characterized by two continuous states, according to the continuity of the current inductor i_L . Keys S1 and S2 operate alternately and made perfect.

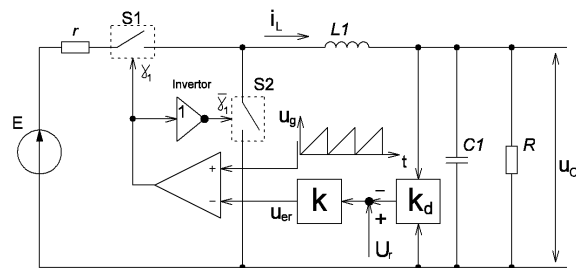


Fig. 1. The electrical circuit system with two continuous states.

Equations corresponding to (1) and describing the operation of the circuit shown in Figure 1 are as follows:

$$\begin{cases} \begin{pmatrix} \frac{di_L}{dt} \\ \frac{du_C}{dt} \end{pmatrix} = A(\gamma_1) X + B(\gamma_1) = \begin{pmatrix} -\frac{\gamma_1 r}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{pmatrix} \begin{pmatrix} i_L \\ u_C \end{pmatrix} + \begin{pmatrix} \frac{\gamma_1 E}{L} \\ 0 \end{pmatrix}, \\ F(u_{er}, u_g) = u_{zw} = U_M(t - nT)/T - u_{er} = 0, \\ u_{er} = k(u_r - k_d u_C), \\ \gamma_1(u_{zw}) = \begin{cases} 0, & u_{zw} < 0, \\ 1, & u_{zw} \geq 0; \end{cases} \end{cases} \quad (2)$$

where: E - power supply voltage; R, r - the load resistance and source impedance, respectively; L, C - linear inductance and capacitance, k_d - reduction coefficient of output voltage supplied to the control system; u_C, u_r, U_M - respectively the voltage on capacity, foot and amplitude with a period T; u_{er} - k times amplified error signal .

3. Simulation parameters

Process modelling is carried out with the software written in C++ and a computer with a microprocessor

Intel ® Core™ 2 Duo 2 GHz, RAM 2 GB. The calculation of differential equations is performed numerically using the Runge-Kutta fourth-order accuracy with a fixed step $\Delta t = 10^{-7}$ sec. Selecting a step is carried out with the aid of simulation Δt implemented empirically. At relatively large values of Δt it is not possible to obtain a long chaotic process because of the low accuracy of the coordinates of the point of intersection of the error signal and the unfolding tension. For small values of Δt the simulation time and memory usage can not solve the problem. Therefore, the step $\Delta t = 10^{-7}$ sec was chosen, providing the possibility for calculating a chaotic process in the range of necessary duration due to the appropriate accuracy of the solution. The average duration of the simulation is 600 seconds. The algorithm of modelling is shown in Figure 2.

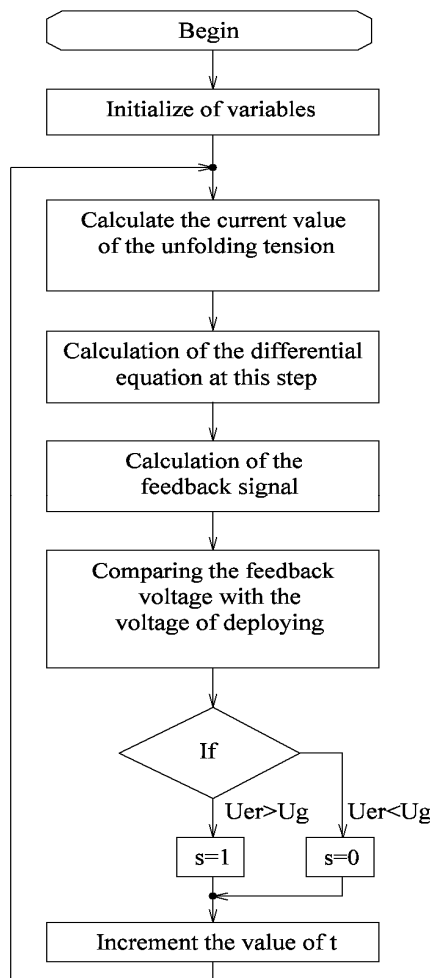


Fig. 2. Simulation algorithm.

Let's note that the modelling has been conducted for the unfolding of various forms of voltage: the piecewise harmonic, exponential and piecewise-power periodic, and the fundamental impact on the nature of the process is not provided.

4. Simulation results

For circuit parameters: $R = 100$ Ohms; $r = 0.1$ Ohm, $L = 0.1$ H, $C = 10^{-6}$ F, $E = 1000$ V, $U_r = 10$ V, $U_M = 10$ V, $T = 0.001$ s, $k_d = 0.01$, $k = 18$, the roots of p_1, p_2 characteristic polynomial of A - are valid.

If $\gamma_1 = 0 - p_1 = -8873.0, p_2 = -1127.0$, while at $\gamma_1 = 1 - p_1 = -8872.8, p_2 = -1128.2$, the following dependence $u_{er}(t)$ (Fig. 3) has been obtained, as well as maps $\tau_{n+1} = f(\tau_n)$ (Fig. 4) and $e_{n+1} + \tau_{n+1} = f(e_n + \tau_n)$ (Fig. 5) where τ_n - the moment of transition γ_1 from "1" to "0" for the voltage unfolding period $0 < \tau_n < T$; e_n - integer periods of the unfolding tension embedded in one Chaotic oscillation; $n = 1, 2, 3, \dots$

For given parameters of the system, there are two alternating chaotic process X1 and X2, with the characteristic for each integer fractal sequences, n_{X1} and n_{X2} , where n_{Xi} - integer periods of the unfolding tension embedded in the chaotic interval Xi . Chaotic Interval is the interval on the time axis located between the start point (first interval on the intersection of u_{er} and u_g) and the point of failure (the latter in the interval and the intersection $u_{er} u_g$) generating a random process $Xi, i = 1, 2, 3$. To determine the intervals of the processes X1 and X2 on the time axis let's use the following procedure. Let's construct a map $e_{n+1} + \tau_{n+1} = f(e_n + \tau_n)$, and highlight the curves, which stand for the main processes, curves 1 'and 2' (Fig. 5), and curves 1 and 2, through which the transition occurs between the main curve of the process.

Then, considering the time dependence u_{er} (Fig. 3) swings are to be added to the process by which the curves lie on the switching point corresponding to this vibration.

Time dependence of u_{er} (Fig.3) is a non-periodic oscillation whose amplitudes vary and are not repeated in time.

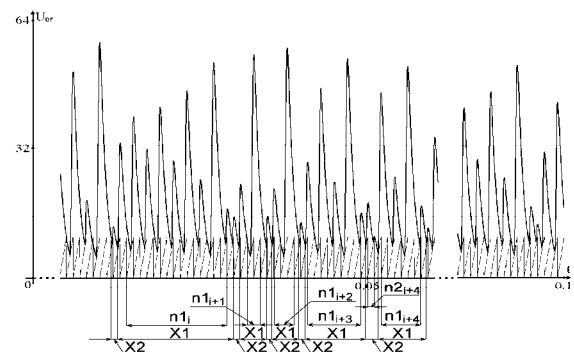


Fig. 3. Alternating sequence of random processes, X1 and X2 with integer fractal sequences $n1_i$ and $n2_i$, respectively.

Mapping $\tau_{n+1} = f(\tau_n)$ for processes X1 and X2 (Fig. 4) represents two curves 1 and 2. These curves correspond to the processes X1 and X2, respectively. Points A and B (Fig. 4) are the points to which the system tends, but they are unstable.

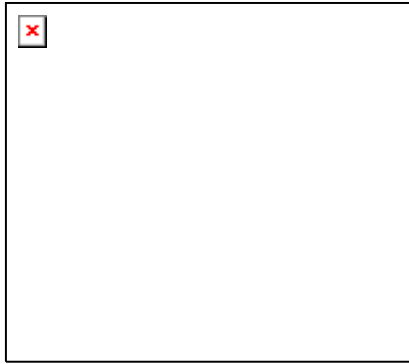


Fig. 4. Map $\tau_{n+1} = f(\tau_n)$

Mapping $e_{n+1} + \tau_{n+1} = f(e_n + \tau_n)$ for processes X1 and X2 (Fig. 5) consists of four curves 1', 1'' and 2', 2''. These curves correspond to the processes X1 and X2, respectively. Points A and B (Fig. 5) are the points to which the system tends, but they are unstable.

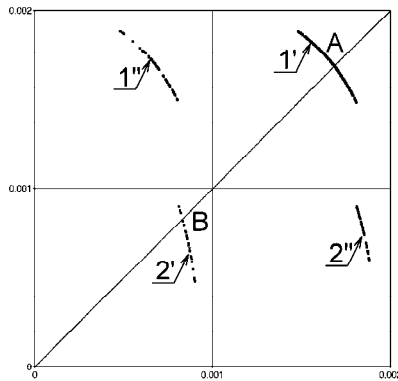


Fig. 5. Map $e_{n+1} + \tau_{n+1} = f(e_n + \tau_n)$

Table 1

X1	...	n1	n1	n1	n1	n1	n1	n1	n1	n1	n1	n1	n1	n1	n1	n1	n1	n1	n1	...
$\delta \approx 0.75$...	7	5	23	9	11	7	19	1	5	17	13	3	7	3	7
X2	...	n2	n2	n2	n2	n2	n2	n2	n2	n2	n2	n2	n2	n2	n2	n2	n2	n2	n2	...
$\delta \approx 0.27$...	0	2	1	0	1	0	0	1	0	1	1	0	0	0	0	0	0

The common feature of the described processes is the presence of integer sequences, characteristic for each of the intermittent chaotic processes, as illustrated in Table 1. The table shows the integer sequence $n1_i, n2_i$ ($i = 1, 2, 3, \dots$) for the chaotic X1 and X2, respectively. For the process X1 an integer number consists of numbers which are either simple, or multiples of 3 or 5, and an integer number of X2 contains the number zero and the prime numbers. The integer sequences obtained are fractal and non-recurrent. The constant process of a deterministic component τ_n , is calculated with the formula $\delta = \frac{\tau_n - \tau_{n-1}}{\tau_{n+1} - \tau_n}$ for X1 equal to $\delta \approx 0.75$, and X2 equal to $\delta \approx 0.27$.

With an increase in the gained k , the number of intermittent chaotic processes increases. The number of alternating processes in complex-conjugated roots of the characteristic polynomial of A increases significantly, preserving their character.

4. Conclusions

A new type of chaotic oscillations consisting of alternating chaotic processes has been discovered and described. Each of these processes consists of integral non-periodic fractal sequences, as well as a deterministic chaotic component with its inherent continuous process. It has been shown that the observed processes represent a new class of physical object motions.

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ДОСЛІДЖЕННЯ СТРИБКОПОДІБНИХ ХАОТИЧНИХ ПРОЦЕСІВ У КУСКОВО-НЕПЕРЕРВНИХ СИСТЕМАХ

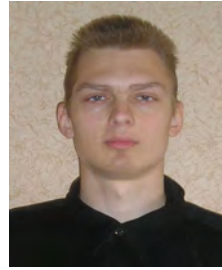
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У статті розглянуто структура хаотичних характеристик кусково-неперервних систем та показано, що розглянуті процеси представляють новий клас рухів фізичних об'єктів, що складаються зі змінних хаотичних коливань, кожне з яких складається з інтегральної аперіодичної фрактальної та детерміністської послідовності хаотичних компонент, притаманних неперервному процесу.



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