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# ELECTRONIC COMMUTATOR WITH PARALLEL CAPACITY STORAGE FOR SWITCHED RELUCTANCE MOTORS

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**Abstract.** A switched reluctance motor (SRM) is simpler and more cost-effective than other low-powered electric motors. The wide circulation of switched reluctance motors is restrained only by their low power indexes. The paper has proposed new energy buffer electrical schemes, which not only prevent power transistors surge voltage, but also reuse the storage power for the forcing a current in the next section, i.e. the energy is saved; additionally, a mathematical model in instantaneous values for this kind of motor has been developed, which is the basis for computation of its characteristics. Examples of computation of dynamic and static characteristics are presented which show that the proposed instantaneous values mathematical model for SRM provides a good basis for computation of its characteristics.

**Key words:** Switched Reluctance Motor; transistor switches; capacity storage; mathematical model**.**

## 1. Introduction

Economical and rational use of material and labour resourses in the branch of electrical machine manufacturing is closely connected with the future developments of electrical machines, i.e. improvements in their quality and longevity as well as with increasing the adaptability to manufacturing process and, therefore dropping, unit costs. Application of electronic switched motors is one of the prospective solutions to such problems.

Brush-less DC motors are electrical machines, in which the brush-collector unit is replaced by the semiconductor commutator – inverter controlled by signals $f(\theta)$ from rotor position sensor (RPS) [1].

One of the simplest by construction, more practically feasible and reliable is a switched motor with a salient-pole stator, concentrated coils of its winding and a toothed passive rotor [2]. Such motor is simpler, cheaper and more technological than the simplest of electrical machines – asynchronous, and its control characteristics do not concede to direct current motors - that results in the wide use of these electrical machines with electric drives where high consumer parameters are required.

In the simple case a switched reluctance motor (SRM) comprises an electromechanical converter (EMC) + rotor position sensor + electronic converter (EC). In [3] some new kinds of EMC construction of SRM are represented but for practical application as most rational it is possible to count constructions.

Many popular low-powered drives use switched motors with permanent excitation and transistor commutators. But the switched motors with permanent magnets on a rotor have such disadvantages as complexity of construction and raised manufacturing costs [3].

More advantageous for such applications seems to be a motor with silent poles and windings on a stator and with a toothed passive rotor. Such an electromechanical converter is very rugged and besides it is simpler and cheaper than the simplest asynchronous motor. Also the regulating properties of switched motors on the basis of this converter are excellent, similar to these of the DC collector motors. A significant amount of publications in technical periodicals and increased interest of electrical engineering companies confirm fast-growing popularity of SRM. Technological effectiveness, work stability and good control characteristics caused wide application of these electrical machines in electric drives, especially in those of high consumer requirements [10].

The wide circulation of switched reluctance motors are restrained only by their low power indexes. They are conditioned by the necessity of dissipation of electromagnetic field energy during the current commutation in order to protect transistor switches from surge voltage. Some new circuit solutions of transistor inverters for SRM were offered in [4]. They enable not only to protect the power transistors from too high voltage, but also to reuse the above mentioned energy for forced a current in the next stator section, i.e. energy saving.

For improved use of the SRM the active zone of a signal sector of RPS should be greater than $120^0$. Thus the amount of sections connected to the power source differs; there is significant pulsing of SRM feed current. It is undesirable to use SRM in low-power controlled electric drives fed from a source of limited power [5]..

## 2. Electronic Commutators with Parallel Capacity Storage

A semiperiodic commutator with additional parallel capacitor [6] permits to improve power indexes and to reduce current pulsing of supplying source of a switched motor with a passive rotor.

### *2.1. Commutator with parallel capacity storage and individual accelerating switches*

The basic electric scheme of a switched reluctance motor with a parallel energy buffer is given in Fig.1.

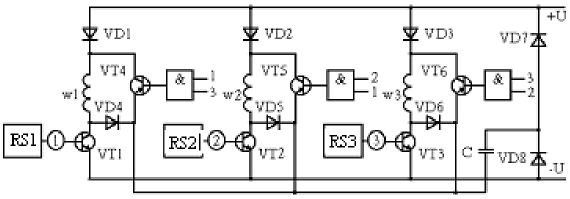


*Fig. 1. Transistor commutator with parallel capacity storage and individual accelerating circuits*

### *2.2. Commutator with parallel capacity storage and a common accelerating switch*

Similarly it is possible to increase reliability of SRM with parallel capacity storage (CS) using one common switch and one diode instead of separate transistor switches and diodes for each section. The resulting scheme is shown in Fig.2. In this electronic commutator (EC) the control of the common transistor switch *VT4* occurs as in the previous case, by signals from the scheme which detects the coincidence of signals of adjacent channels of RPS.

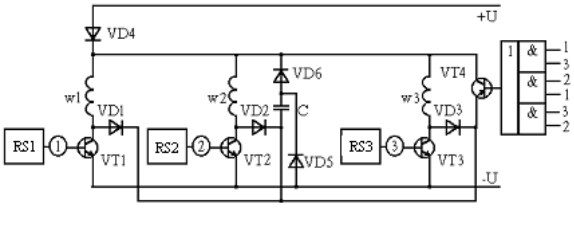


*Fig. 2. SRM with the parallel buffer and common accelerating circuit*

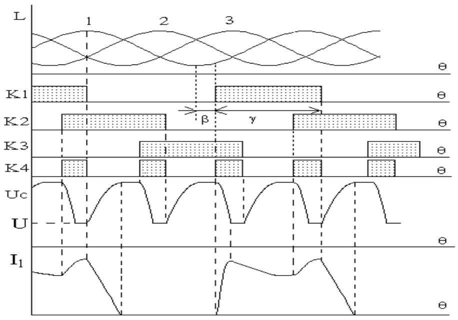


*Fig. 3. Section inductances, work diagram of transistor switches, capacitor voltage and current of one section of SRM with a parallel capacity storage and common accelerating circuit*

The capacitor charge in this case is the same as in SRM with serial CS and it occurs through the diodes *VD1 - VD3* and *VD6*. There is a forced drop of the section current. The capacitor is charged to the voltage, which is higher than the voltage of the supplying source.

In certain rotor positions there is simultaneous turn-on of one force switch (*VT1 ÷ VT3)* and of the accelerating switch *VT4*. Two sections of armature winding are now connected to the capacitor. The diode *VD4* is at this time switched off by capacitor voltage. The discharging of the capacitor follows (see Fig. 3). When its voltage reaches the level of supplying voltage the diode *VD4* opens again and both sections obtain energy from the power source.

## 3. Mathematical Model of SRM with a Parallel Capacity Storage

For synthesis and analysis of electronic commutated motor systems it is necessary to develop adequate mathematical models of electromechanical processes in such systems. Therefore, improvement in already existing and development of new mathematical models of known and new switched motors with passive rotors are very actual tasks [7].

### *3.1. Assumptions*

For creating of the mathematical model of switched motor we will accept next assumptions [8], which not distorting the real physical processes will permit to receive clear mathematical dependences concerning the energy transformation in it:

- ✓ the internal resistance of converter voltage source equal to zero;
- ✓ no magnetic coupling between sections;
- ✓ electronic power switches of the commutator with infinite resistance in off-state, negligible turn on and turn-off times and instantly section commutations without switch losses;
- ✓ the diodes in a conducting state represented by a typical mathematical model (volt-ampere characteristic approximated by expression $i_\partial = I_0\left(e^{b\cdot\Delta U_\partial} - 1\right)$ and the diode voltage drop by $\Delta U_\partial = \ln\frac{i_\partial + I_0}{I_0}/b$ ; where $I_0$ *and* $b$ - inverse current of the diode and coefficient of temperature potential) and with infinite resistance in the off state;
- ✓ magnetic characteristic of magnetic insulated section represented by expression [3]:

$$\psi(\theta,i)=\left[\psi_{10}-\psi_{1t}\cdot Sin(\theta/2)\right]\cdot i+\psi_y\cdot Sin(\theta/2)\cdot\left[1-e^{-a\cdot i\cdot Sin(\theta/2)}\right]$$

- ✓ concentrated parameters of stator windings and SRM scheme according to the Fig.1

### *3.2. Modelling of iron losses*

The alternating magnetic flux creates in the iron core an electromotive force, which is the cause of eddy (Foucault) currents and therefore losses in this core. The cross-sectional of magnetic circuit can be considered as consisting of elementary closed loops, which correspond to shortly circuited coils penetrated by magnetic flux. Let's mark them in scheme of SM section as a branch $L_\sigma \& R_s$, which is parallel connected to the magnetizing branch. The iron resistance in connection with eddy currents can be calculated, with precision sufficient for engineering practice by the equation $R_s = \frac{E^2}{\Delta P_s}$, where $E = 4.44 \cdot f \cdot w_z \cdot B \cdot s$ and $\Delta P_s = P_0 \cdot \gamma_s \cdot s \cdot l_{«} \cdot B^2 \cdot \left(f / f_0\right)^2$. For calculation of one section $R_s$ we receive the equation:

$R_s = 4{,}9 \cdot 10^4 \cdot \frac{w_z^2 \cdot S \cdot q}{p_0 \cdot \gamma_s \cdot l_m}$, where $p_0, \gamma_\mathrm{s}, \mathrm{S}, \mathrm{l_m}, \mathrm{w_z}, \mathrm{q}$ are specific losses in iron, density of a magnetic material, length of a magnetic force line, amount of winding coils per one tooth and amount of stator teeth per one section accordingly.

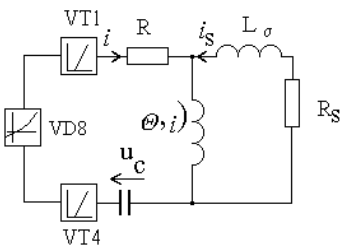
The leakage inductance $L_\sigma$ of eddy current circuit is insignificant and can be neglected. However for ensure of stability of the differential equations solution (numerical methods) is offered the acceptation of the values: $L_\sigma \cong \frac{\Delta t}{12} \cdot R_s$, where $\Delta t$ - an integration step of the differential equations.

*3.3. Digitization of the scheme connection of SRM section*

In correspondence with the accepted assumptions, we can consider electrical each section of m-sectional SM separately and to link them only through an electromagnetic torque, created by them and acting on a rotor.

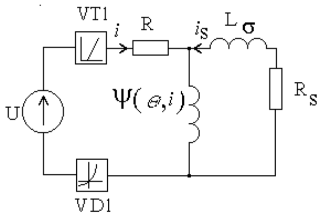
According to the algorithm of SRM-work and to accepted assumptions the scheme of activated structure and the voltage equations will discretely vary with a change of the angle $\theta$ [9] :

1. The transistor switches *VT1* and *VT4* are on, capacitor *C* is charged. The diode *VD1* is turned off by reverse capacitor voltage. Section *w1* is feeded with the energy storaged in the electrical field of the capacitor:



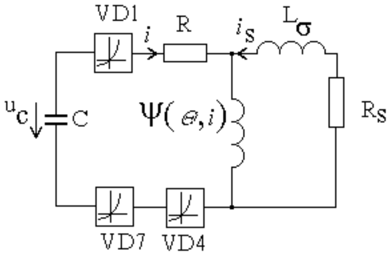
$$\begin{cases} R \cdot i + 2 \cdot \Delta U_T + \Delta U_\partial + \dfrac{d\psi(\theta,i)}{dt} = u_c \\ C \dfrac{du_c}{dt} = i \\ R_s \cdot i_s + L_\sigma \cdot \dfrac{di_s}{dt} + \dfrac{d\psi(\theta,i)}{dt} = 0 \end{cases} \tag{1}$$

2. The transistor switch *VT1* is in on-state, the capacitor *C* is partial discharged. Section *w1* receives feeding from the power source through the diode *VD1* and the switch *VT1*:



$$\begin{cases} R \cdot i + \Delta U_\partial + \Delta U_T + \dfrac{d\psi(\theta,i)}{dt} = U \\ R_s \cdot i_s + L_\sigma \cdot \dfrac{di_s}{dt} + \dfrac{d\psi(\theta,i)}{dt} = 0 \end{cases} \tag{2}$$

3. Transistor switches *VT1* and *VT4* are off. The energy of the section magnetic field charges the capacitor through the diodes *VD4*, *VD7* and *VD1*:



$$\begin{cases} R \cdot i + 3 \cdot \Delta U_\partial + \dfrac{d\psi(\theta,i)}{dt} = -u_c \\ C \dfrac{du_c}{dt} = -i \\ R_s \cdot i_s + L_\sigma \cdot \dfrac{di_s}{dt} + \dfrac{d\psi(\theta,i)}{dt} = 0 \end{cases} \tag{3}$$

The additional formal coefficient *K1* and *K2* enable to realize a generalized entry of nonlinear system of the differential equations (NSDE) (1) - (3):

$$\begin{cases} R \cdot i + \dfrac{d\psi(\theta,i)}{dt} = \left(U + \Delta U_d\right) \cdot \left(K1 - K2\right) + u_c\left(K1 + K2 - 1\right) - \\ - \Delta U_T\left(K1 + K2\right) - \Delta U_d\left(3 - K1 - K2\right) \\ C \dfrac{du_c}{dt} = i\left(1 - K1 - K2\right) \\ R_s \cdot i_s + L_\sigma \cdot \dfrac{di_s}{dt} + \dfrac{d\psi(\theta,i)}{dt} = 0 \end{cases} \tag{4}$$

### *3.4. Model of power transistor switches*

The modelling of power transistor switches is realized with assumptions accepted in III.1 (i.e. transient process of their turning on and turning off happen instantly, the resistance in off-state is infinite). The volt-ampere characteristic of a saturated switch is described by expression:

$$\Delta U_T = U_{KE} = U_{KE.0} + R_{KE.\text{н}A„} \cdot i \ ,$$

where: $U_{KE.0}$ *and* $R_{KE.\text{н}A„}$ result from the transistor data.

The state of transistor switches in EC depends on the mutual rotor and stator position, i.e. the transistors are controlled by signals of the rotor position sensors. Let's assume in the correspondence to the condition of these switches a formal coefficient $K_j$, which gains a value "1" - if a switch is in on-state, and "0" - if a switch is in off-state and where "j" is the number of a switch.

The values of formal coefficient $K_j$ depend on rotor position and can be calculated according to the following switching function:

$$K_j = 1 \ \ if \ \beta + 2\cdot\left(N_j - 1\right)\cdot\pi + \left(j-1\right)\cdot 2\cdot\frac{\pi}{m} \langle \theta \le \beta + 2\cdot\left(N_j - 1\right)\cdot\pi + \left(j-1\right)\cdot 2\cdot\frac{\pi}{m} + \gamma \qquad (5)$$

$K_j$ = 0 for all other values,

where: $N_j = \dfrac{\theta + \pi + \left(j-1\right)\cdot 2\cdot\dfrac{\pi}{m}}{2\cdot\pi} + 1$ - number of a phase for appropriate section, but if $u_c$ = U then $K_{j+3}$ = 0.

According to a principle of drive operation, it is possible to use any control strategy of the switches *VT4 - VT6*. It is however important, that they should be off at the moment of a current cut-off in any section. One of the elementary control strategies of the accelerating switches is a mode, when the control signals are formed by a logic multiplication of signals of contiguous channels of rotor position sensors (Fig.3). In a m-sectional motor, unlike a three-sectional, the multiplied signals of no contiguous channels, and the choice of channel pairs depends on an amount of motor sections and on value of commutation interval γ:

$$K_{j+m} = K_j \cdot K_i \qquad (6)$$

where: $i = m - l + j$

if $2\cdot\left(l+1\right)\cdot\pi / m \ge \gamma \ge 2\cdot l\cdot\pi / m$, $l$ = 1,2,3,4;

and *i --> i-m* if *i > m.*

For the SRM with common accelerating circuit according to Fig.2 the formal coefficient can be calculated as:

$$K_{m+1} = \sum_{j=1}^{m} K_j \cdot K_i$$

### *3.5. Mathematical model of SRM with parallel capacity storage*

Taking into account above-mentioned, it is possible to present the NSDE, which describes electromechanical processes in SRM with parallel capacity, as:

$$\left\{\begin{array}{l}
\dfrac{di_j}{dt} = \left[u_j + R\cdot i_j + \dfrac{A_j}{L_\sigma}\cdot\left(u_j - R\cdot i_j + R_s\cdot i_{sj}\right) - B_j\cdot\omega\right] / A_j; \\[2ex]
\dfrac{di_{sj}}{dt} = -\left(R_s\cdot i_{sj} + u_j - R\cdot i_j\right) / L_\sigma; \\[2ex]
\dfrac{du_{cj}}{dt} = \left(1 - K_j - K_{j+m}\right)\cdot\dfrac{i_j}{C}; \\[2ex]
\dfrac{d\omega}{dt} = \left\{\sum_{j=1}^{m}\left[\begin{array}{l}\dfrac{z_r}{2}\cdot\left(i_j + i_{sj}\right)\cdot Cos\theta_{pj}\cdot\psi_{-}\cdot \\ \left(1 - e^{-a\cdot\left(i_j + i_{sj}\right)\cdot Sin\theta_{pj}} - \dfrac{\psi_{1t}\cdot\left(i_j + i_{sj}\right)}{2\cdot\psi_{-}}\right)\end{array}\right] - M_C\right\}\cdot\dfrac{z_r}{J}; \\[2ex]
\dfrac{d\theta}{dt} = \omega; \\[2ex]
\partial e \ \ u_j = \left(U + \Delta U_\partial\right)\cdot\left(K_j - K_{j+m}\right) + u_c\cdot\left(K_j + K_{j+m} - 1\right) - \\
- \Delta U_T\cdot\left(K_j + K_{j+m}\right) - \Delta U_\partial\cdot\left(3 - K_j - K_{j+m}\right); \\
\qquad A_j = \psi_{10} - \psi_{1t}\cdot Sin\theta_{pj} + \psi_{-}\cdot a\cdot Sin^2\theta_{pj}\cdot e^{-a\cdot\left(i_j + i_{sj}\right)\cdot Sin\theta_{pj}}; \\
\qquad B_j = \dfrac{Cos\theta_{pj}}{2}\cdot\left[\begin{array}{l}\psi_{-} - \psi_{1t}\cdot(i_j + i_{sj}) - \psi_{-}\cdot e^{-a\cdot\left(i_j + i_{sj}\right)\cdot Sin\theta_{pj}}\cdot \\ \cdot\left(1 - a\cdot(i_j + i_{sj})\cdot Sin\theta_{pj}\right)\end{array}\right]; \\[2ex]
\qquad \theta_{pj} = \dfrac{\theta}{2} - \left(j-1\right)\cdot\dfrac{2\cdot\pi}{m};
\end{array}\right. \qquad (7)$$

The solutions of this NSDE have been obtained using the fourth order Runge-Kutta method with constant integration step [9].

### *3.6. Application of the method of the differential equations inverting for the mathematical model of SRM*

Since the angle *θ*, section currents $i_j$ and voltage on the capacitor storage $u_c$ are time functions, then for NSDE- integration there is a necessity of using of iterative methods. It enables the exact definition of turning on and turning off moments of power transistor switches, as well as of the moment, when the section current reaches zero value, and also when capacitor voltage achieves a value of power supply voltage. However isn't quite simple to hit exact the moment of commutation with use of integration methods higher order. Therefore more often one applies approximate value of the commutation moment, using iterative methods and accepting some admissible error. Such way has two shortages. At first, there is no mathematically well-grounded criterion for the definition of an admissible error of the commutation moment and, secondly, even for the approximate identification of commutation moment the further step subdivisions are necessary.

The most natural solution of a problem of commutation moment searching is the application of “inverting NSDE method” [5]. The essence of this method for given task is, that near of turning on or turning off moment of any power switch of the EC, in the case of his state changing during the following integration step, as the independent variable we accept not time *t* but angle $\theta$. The integration step can be now defined using known value of an angle $\theta$ (i.e.$\beta$, see Fig.4) minus the value $\theta_i$ found in the last integration step.

The NSDE, which describes variable modification in function of an angle $\theta$, we obtain using a rule of derivation of composite functions:

$$\frac{di_j}{d\theta}=\frac{di_j}{dt}\cdot\frac{dt}{d\theta};\quad \frac{\mathrm{d}i_{\mathrm{sj}}}{\mathrm{d}\theta}=\frac{\mathrm{d}i_{\mathrm{sj}}}{\mathrm{d}t}\cdot\frac{\mathrm{d}t}{d\theta};\quad \frac{\mathrm{d}u_{\mathrm{cj}}}{\mathrm{d}\theta}=\frac{\mathrm{d}u_{\mathrm{cj}}}{\mathrm{d}t}\cdot\frac{\mathrm{d}t}{d\theta};$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}\theta}=\frac{d\omega}{dt}\cdot\frac{dt}{d\theta};\qquad \frac{\mathrm{d}t}{\mathrm{d}\theta}=\frac{1}{\omega};\qquad \frac{\mathrm{d}\theta}{\mathrm{d}\theta}=1.$$

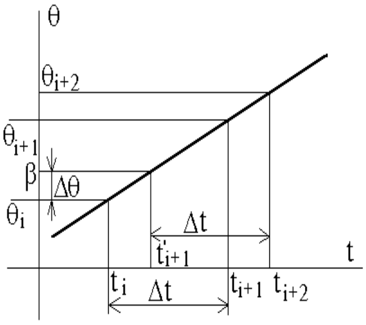


*Fig. 4. Graphical explanation of the NSDE inversion method for exact estimation of the of turn on moment of a power switch*

The derivations $\frac{di_j}{dt}$, $\frac{du_{cj}}{dt}$, $\frac{d\omega}{dt}$ in the above mentioned equations can be calculate according to (7).

The integration procedure after an angle $\theta$ is the same as after the time *t*. The time is in this case an angle function. As a result of this integration step after an angle an exact value of commutation moment $t'_{i+1}$ of section current have been obtained and further we can continue to integrate NSDE after time, by inverting it in a previous state.

In exactly the same way the NSDE is inverted, if the current of any section during the integration step changes the sign, only using this section current as the independent value instead of time.

## 4. Results of Computer Simulation of Electromechanical Processes in SRM with Parallel Capacity Storage

In the Fig.5 the example of simulation of a three-section switched reluctance motor with the parallel capacity energy buffer (quasi-steady state value of section current and capacitor voltage) is presented.

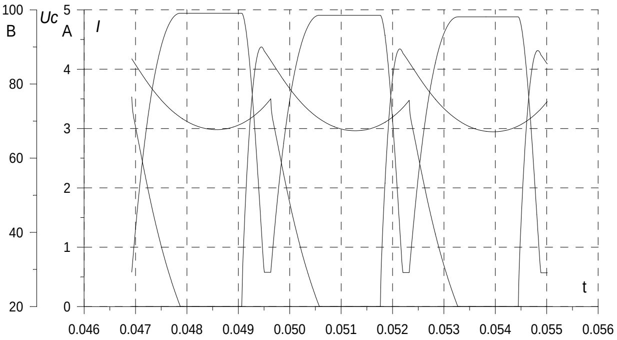


*Fig.5. Quasi-steady state of section current and capacitor voltage of SRM with a parallel capacity storage (computed values)*

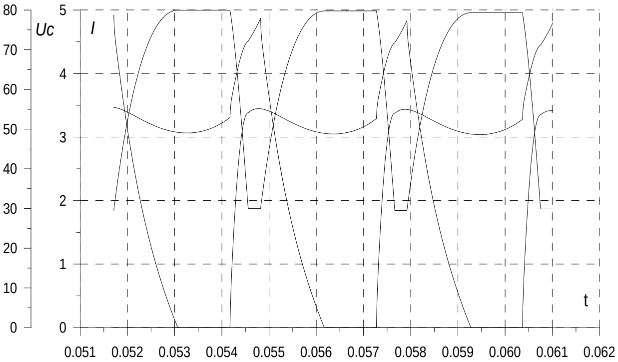


*Fig.6. Quasi-steady state of capacitor voltage and section current in three-sections SRM with the parallel energy buffer and common accelerating switch (computed values)*

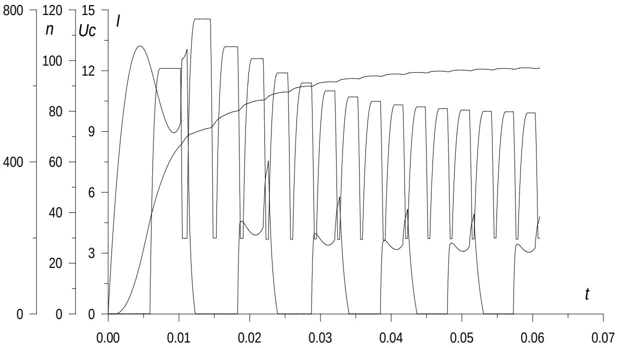


*Fig.7. Simulation results for starting of SRM with the parallel capacity storage (current of one section, capacitor voltage and rotation speed)*

A computed example of starting transient of three-section switched reluctance motor with the parallel energy buffer (current of one section, electromagnetic torque, rotation speed) is given in the Fig. 8.

## 5. Conclusions

The comparisons of simulation results with oscillograms of the real system, and also average values of a torque, rotation speed and composite losses for examined SRM with nominal torque from 0.05 up to 20 Nm show, that the divergences don`t exceed 5 %, that confirms the adequacy of a mathematical model to a physical sample.

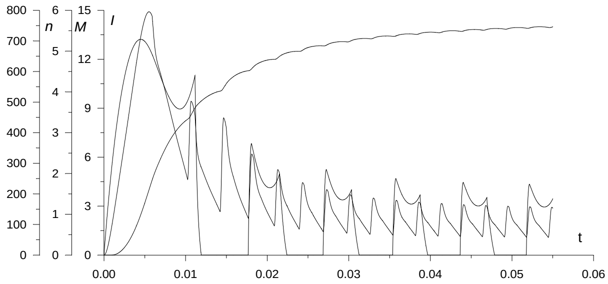


*Fig. 8. Computed values of section current, electromagnetic torque and rotation speed of three-section SRM with parallel energy buffer during its starting*

The switched reluctance motor is simpler, more suitable for efficient manufacturing and cheaper than other known low powered electric motors. As the presented dynamic and static characterisics show, the proposed average values mathematical model for SRM provides a good basis for computation of its characteristics.

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## ЕЛЕКТРОННИЙ КОМУТАТОР З ПАРАЛЕЛЬНИМ ЄМНІСНИМ НАКОПИЧУВАЧЕМ ДЛЯ ВЕНТИЛЬНОГО РЕАКТИВНОГО ДВИГУНА

**Василь Ткачук, Маріуш Клита**

Вентильні реактивні двигуни є простішими та більш ефективними ніж інші електричні двигуни малої потужності. Широке застосування вентильних реактивних двигунів стримується тільки через їхні низькі енергетичні показники. В статті запропоновані нові електричні схеми з буферами енергії, які не тільки захищають силові транзистори комутатора від перенапруг під час вимкнення секції, але також використовують запасену в магнітному полі енергію для форсування струму в наступній секції, тобто економиться енергія і математична модель для миттєвих значень цього типу двигуна, яка є базою для розрахунку його характеристик. Представлено приклади розрахунку динамічних та статичних характеристик, які показують, що запропоновані математичні моделі є доброю базою для розрахунку характеристик таких двигунів.

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