

# THEORETICAL BASES OF ASYMPTOTIC STABILITY ASSESSMENT OF LINEAR PARAMETRIC CIRCUITS ACCORDING TO FREQUENCY METHOD

Yuriy Shapovalov, Bohdan Mandziy

Lviv Polytechnic National University

shapov@polynet.lviv.ua

**Abstract.** The possibility of assessment of asymptotic stability of linear parametric circuits according to the frequency symbolic method and approximation of transfer circuit function performed by truncated Fourier series has been showed. Assessment of asymptotic stability has been carried out according to normal transfer function but not bifrequency as usual. Two examples of assessment of asymptotic stability of parametric amplifiers are given.

**Keywords:** linear parametric circuit, assessment of asymptotic stability, symbolic analysis.

## 1. Introduction

The linear parametric circuit of time  $t$  is characterized by impulse transfer function  $w(t, \xi)$  as a circuit response to delta-impulse given at the time period  $\xi$ . The function of two variables  $w(t, \xi)$  is often used in the form of function of one variable on condition that the other variable is considered the parameter. At the same time the researcher should deal with so-called conjugate impulse reaction when the parameter is  $t$  or normal impulse reaction when the parameter is  $\xi$  [1]. This condition divides the sums of research of linear parametric circuits into two classes. So, the sums connected with the stability analysis in the supervision period [1] are done with the help of normal impulse reaction and according to Laplace their representations are normal parametric transfer functions  $W(s, \xi)$  but the sums connected with the transfer of signals are done with the help of conjugate impulse reactions and their representations according to Laplace that are conjugate parametric transfer functions  $W(s, t)$ . The assessment of asymptotic stability is carried out according to so-called bifrequency transfer function  $W(s, r)$  [1,2], that represents the result of double application of Laplace transformation to function  $w(t, \xi)$ .

The article shows the possibility of assessment of asymptotic stability of linear parametric circuit by parametric transfer function  $W(s, \xi)$  that is formed with much less computational burden than  $W(s, r)$ .

## 2. The purpose of the article

[1] shows that spectral research methods of stability of linear circuits with constant parameters which are

usually used by researchers can be transferred to linear circuits with variable parameters but with some peculiarities. We shall consider these peculiarities and their application. So, this article deals with the problem of assessment of stability of linear parametric circuits performed by analysis of frequency symbolic method [3].

This problem can be solved with the help of transfer function of poles that is quite frequently and usually used for the stability assessment carried out by symbolic analysis of linear circuits with constant parameters.

To reach this aim the following criteria of stability of linear parametric circuit given in [1] are used.

### Criterion 1.

Linear parametric circuit in the given observation interval  $T$  is stable if:

$$\int_{\xi}^{\infty} |w(t, \xi)| dt < \infty, 0 < \xi < T. \quad (1)$$

This leads to the following definition of stability: a linear system with variable parameters is stable in the interval only when its normal parametric transfer function  $W(s, \xi)$  does not have poles in the right half-plane and on the unreal axis of complex plane  $s$  by all  $\xi$ , which are situated in the interval under consideration.

### Criterion 2.

Linear parametric circuit is stable asymptotically  $0 < \xi < \infty$  if the integral

$$\int_0^{\infty} \int_0^{\infty} |w(t, \xi)| dt d\xi \quad (2)$$

is absolutely convergent. It demands the determination of all analytical infringements of bifrequency transfer function  $W(s, r)$  on the plane  $\rho\sigma$  and making so-called convergence characteristic  $\rho = \chi(\sigma)$  and area  $D_l$  on this basis [1,2]. If area  $D_l$  includes points with  $\sigma < 0$ , the circuit with such characteristics  $\rho = \chi(\sigma)$  is asymptotically stable.

The possibilities of application of criteria (1) and (2) for assessment of the stability of linear parametric circuits when analysed by frequency symbolic method

are considered and the peculiarities of such application are determined.

### 3. Main body

To calculate function  $W(s,t)$  in [3] its approximation  $\mathbb{W}(s,t)$  in the form of truncated Fourier series is successfully used:

$$\mathbb{W}(s,t) = W_0(s) + \sum_{i=1}^k [W_{ci}(s) \cos(i\Omega t) + W_{si}(s) \sin(i\Omega t)] \quad (3)$$

or in complex form

$$\mathbb{W}(s,t) = W_0(s) + \sum_{i=1}^k [W_{-i}(s) \cdot \exp(-j \cdot i \cdot \Omega \cdot t) + W_{+i}(s) \cdot \exp(+j \cdot i \cdot \Omega \cdot t)] \quad (3a)$$

where  $W_0(s)$ ,  $W_{ci}(s)$ ,  $W_{si}(s)$  and  $W_{\pm 0}(s)$ ,  $W_{-i}(s)$ ,  $W_{+i}(s)$  - independent on time  $t$  fractionally rational functions of complex variable  $s$ ,  $k$  - number of harmonics in the series,  $\Omega = 2\pi/T$ ,  $T$  - period of parameter change of parametric element of the circuit under the influence of pumping signal.

Such approximation has the following positive calculation features:

1. it provides unique simplicity of determination of functions  $W_{\pm 0}(s)$ ,  $W_{-i}(s)$ ,  $W_{+i}(s)$  or  $W_0(s)$ ,  $W_{ci}(s)$ ,  $W_{si}(s)$  in the symbolic form due to the fact that it leads to doing the system of linear algebraic equations [3];
2. it allows to get exact expression for  $\mathbb{W}(s,t)$  (from the point of view of methodic mistake) in any way by increasing the number of  $k$  included in series (3) terms [5];
3. it forms the expression  $\mathbb{W}(s,t)$  which is much more compact than the expressions received by the other, for example, approximate methods from [6];
4. it proved its high efficiency and convenience during the further research of linear parametric circuits [3,5] by researching the function  $W(s,t)$  through the research of its approximation  $\mathbb{W}(s,t)$ .

The task of stability determination as mentioned above demands the previous determination of functions  $W(s,\xi)$  or  $W(s,r)$ . Because of the mentioned positive peculiarities of approximation (3) and (4) the formation of function  $W(s,\xi)$  should be carried out by analogy, for example in the form of trigonometric Fourier series:

$$\mathbb{W}(s,\xi) = W_0(s) + \sum_{i=1}^k [W_{ci}(s) \cos(i\Omega \xi) + W_{si}(s) \sin(i\Omega \xi)] \quad (4)$$

where  $W_0(s)$ ,  $W_{ci}(s)$ ,  $W_{si}(s)$  - fractionally rational functions of  $s$  with the same denominator  $\Delta(s)$  according to frequency symbolic method.

But according to criterion 1 the approximation (4) has a serious drawback when assessing stability. It has turned out that finding the poles of the function  $W(s,\xi)$  defined in the form of trigonometric Fourier series does not lead to the desired aim and it is supported by the following arguments.

#### Argument 1.

At different meanings  $\xi$  in the general case the circuit can be stable or unstable as its impulse transfer characteristic  $w(t,\xi)$  can be falling at some values of  $\xi$  and not falling time function  $t$  at the others.

#### Argument 2.

As the position of poles in function  $W(s,\xi)$  (4) or denominator root of function  $W(s,\xi)$  on the plane  $s$  determines stability or instability of the circuit, the argument shows that the value of these poles (denominator roots) must depend on  $\xi$ .

#### Argument 3.

The approximations (4) and (3,a) have denominators only in expressions  $W_0(s)$ ,  $W_{ci}(s)$ ,  $W_{si}(s)$ ,  $W_{\pm 0}(s)$ ,  $W_{-i}(s)$ ,  $W_{+i}(s)$  which do not depend on  $\xi$ . This means that these denominator roots do not depend on  $\xi$  either.

The content of arguments 1 and 3 leads to the conclusion that according to criterion 1 the assessment of circuit stability with the help of denominator roots of function  $W(s,\xi)$  approximated by Fourier series does not lead to desired result. To solve this problem it is necessary to offer the other approximation of function  $W(s,\xi)$  that by-turn makes its definition not always possible.

But the situation is not hopeless. As approximation (4) has the above mentioned positive calculating parts of approximation (3), it can be used to determine function  $W(s,r)$  to which the criterion 2 can be applied. So, we shall use the dependence given in [1]

$$W(s,r) = \int_0^{\infty} W(s,\xi) e^{-r\xi} d\xi \quad (5)$$

This approach to stability determination according to criterion 2 is reasonable as it is based on the fact that approximation (4) of the function  $W(s,\xi)$  gives quite simple calculation in (5) not having denominators dependable on  $\xi$  in contradistinction to the other known methods of  $W(s,r)$  determination. So, when having approximation of function  $W(s,\xi)$  by expression (4) the determination of the integral in (5) leads to the following expression:

$$W(s,r) = W_0(s) \int_0^{\infty} e^{-r\xi} d\xi + \sum_{i=1}^k [W_{ci}(s) \int_0^{\infty} \cos(i\Omega \xi) e^{-r\xi} d\xi + W_{si}(s) \int_0^{\infty} \sin(i\Omega \xi) e^{-r\xi} d\xi] \quad (6)$$

Taking into consideration that integrals from expression (6) are:

$$\int_0^{\infty} e^{-r\xi} d\xi = \frac{1}{r}, \quad \int_0^{\infty} \cos(i\Omega \xi) e^{-r\xi} d\xi = \frac{r}{r^2 + (i\Omega)^2}$$

$$\int_0^{\infty} \sin(i\Omega \xi) e^{-r\xi} d\xi = \frac{i\Omega}{r^2 + (i\Omega)^2},$$

we get the expression for bifrequency transfer function of the circuit in the form of:

$$W(s, r) = W_0(s) \frac{1}{r} + \sum_{i=1}^k [W_{ci}(s) \frac{r}{r^2 + (i\Omega)^2} + W_{si}(s) \frac{i\Omega}{r^2 + (i\Omega)^2}] \quad (7)$$

According to the criterion of asymptotic stability it is necessary to determine all the analyticity breakings that are determined by the root of this denominator in the expression (7). It can be seen from the expression (7) that its denominator which is designated as  $\Delta(s, r)$  can be in the following form

$$\Delta(s, r) = \Delta(s) \cdot r \cdot \prod_{i=1}^k (r^2 + (i\Omega)^2). \quad (8)$$

The form of expression (8) shows that set of roots of expression  $\Delta(s, r)$  consists of set of roots of polynomial  $\Delta(s)$  of the function  $W(s, \xi)$  and roots:

$$r_0 = 0, r_{1,2} = \pm j\Omega, r_{3,4} = \pm j2\Omega, \dots, r_{2k-1,2k} = \pm jk\Omega \quad (9)$$

As real parts of all roots in (9) equal null they will be presented as straight lines put on each other that merge with the axis  $\sigma$  on the plane  $\rho\sigma$ . The real parts of the polynomial roots  $\Delta(s)$  of (4) in the plane  $\rho\sigma$  will be presented as straight lines parallel to axis  $\rho$  that cross axis  $\sigma$  because of the meaning of real part of its root. The characteristics of convergence  $\rho = \chi(\sigma)$  in this case will look like a right angle, one side of which is on axis  $\sigma$  and the other one is on the vertical line that corresponds to the root  $\Delta(s)$  with the biggest real part and its vertex is situated in the point where this line crosses the axis  $\sigma$ . Plotted in such a way the right angle includes upper right quadrant of the plane  $\rho\sigma$  and as a result it forms sphere  $D_I$ . It is obvious that in this case the sphere  $D_I$  does not include points from  $\rho < 0$  but a) it includes points from  $\sigma < 0$  if the biggest real part among real parts of all roots  $\Delta(s)$  is negative or b) does not include points from  $\sigma < 0$  if the biggest real part among real parts of all roots  $\Delta(s)$  is null or positive. It can be seen that in case (a) the circuit is stable and in case (b) it is unstable. Based on the method of forming sphere  $D_I$  and previous considerations we can draw a conclusion [4]: approximation  $\mathbb{W}(s, t)$  in the form of (4) changes the problem of asymptotic stability of the circuit from the analysis of characteristic of convergence  $\rho = \chi(\sigma)$  of function  $W(s, r)$  to ordinary finding of the biggest real part among real parts of all roots of denominator  $\Delta(s)$  of the normal function of circuit transfer  $W(s, \xi)$ .

The conclusion shown above is drawn for the case of approximation of normal transfer function  $W(s, \xi)$  by trigonometric Fourier series (4). On the other hand as it is shown in [5] in case of effectiveness of frequency

symbolic method the approximation of function  $W(s, \xi)$  performed by exponential Fourier series

$$\mathbb{W}(s, \xi) = W_{\pm 0}(s) + \sum_{i=1}^k [W_{-i}(s) \cdot \exp(-ji\Omega\xi) + W_{+i}(s) \cdot \exp(+ji\Omega\xi)] \quad (10)$$

in comparison with trigonometric series (4) is much more attractive and allows to do the sums on considerably higher levels. In expression (10)  $W_{\pm 0}(s)$ ,  $W_{-i}(s)$ ,  $W_{+i}(s)$  are independent of time  $t$  rational functions of complex variable  $s$  that according to frequency symbolic method have the same denominator that will be designated as  $D(s)$ . Such a fact really exists [5], although the form of Fourier series should not have considerable influence on the course and peculiarities of calculation process. So, if the  $W(s, \xi)$  should be calculated by approximation (10) there is necessity to check the possibility of this approximation in tasks of assessment of asymptotic stability of linear parametric circuits performed by the analysis of frequency symbolic method.

We will check it on the ground of the following considerations. Also we can observe that as the relation between rational function of (4) and (10) has the form:

$$W_0(s) = 2W_{\pm 0}(s), W_{ci}(s) = W_{-i}(s) + W_{+i}(s), \\ W_{si}(s) = j[W_{-i}(s) - W_{+i}(s)],$$

it is obvious that with the same  $k$  their corresponding denominators  $\Delta(s)$  and  $D(s)$  are equal each other. They are marked by one symbol  $\Delta(s)$  respectively.

The function  $W(s, r)$  can be found in the expression (5) with the approximation of function  $W(s, \xi)$  by series (10).

$$W(s, r) = W_{\pm 0}(s) \int_0^{\infty} e^{-r\xi} d\xi + \sum_{i=1}^k [W_{-i}(s) \int_0^{\infty} e^{-ji\Omega\xi} e^{-r\xi} d\xi + W_{+i}(s) \int_0^{\infty} e^{+ji\Omega\xi} e^{-r\xi} d\xi] \quad (11)$$

Taking into account that:

$$\int_0^{\infty} e^{-r\xi} d\xi = \frac{1}{r}, \int_0^{\infty} e^{-ji\Omega\xi} e^{-r\xi} d\xi = \frac{1}{r + ji\Omega}, \\ \int_0^{\infty} e^{+ji\Omega\xi} e^{-r\xi} d\xi = \frac{1}{r - ji\Omega},$$

we get the expression for bifrequency transfer function of the circuit in the following form:

$$W(s, r) = W_{\pm 0}(s) \frac{1}{r} + \sum_{i=1}^k [W_{-i}(s) \frac{1}{r + ji\Omega} + W_{+i}(s) \frac{1}{r - ji\Omega}] \quad (12)$$

According to the criterion 2 of asymptotic stability [1,2] in the expression (12) it is necessary to determine all the analyticity breakings that are determined by the

roots of its denominator. It can be seen from the expression (12) that its denominator is in the form:

$$\Delta(s, r) = \Delta(s) \cdot r \cdot \prod_{i=1}^k [(r - ji\Omega) \cdot (r + ji\Omega)] \quad (13)$$

that equals the expression for  $\Delta(s, r)$  in (8). Equality of the expressions (8) and (13) shows equality of their roots. Equality of the roots evaluated for the denominators  $\Delta(s, r)$  of bifrequency transfer functions  $W(s, r)$  that are determined by approximations (4) and (10) mean that all the conclusions for the first case are also appropriate for the second one. Finally, we can draw quite an important conclusion appropriate for the both approximations of the function  $W(s, \xi)$ .

### Conclusion

Approximation of the function  $\mathbb{W}(s, \xi)$  in the form of trigonometric (4) or complex (10) Fourier series changes the task of assessment of asymptotic stability of the circuit from the analysis of convergence characteristic  $\rho = \chi(\sigma)$  of function  $W(s, r)$  to the ordinary finding of the most real part among real parts of all roots of denominator  $\Delta(s)$  of the normal transfer function of the circuit  $W(s, \xi)$ . In addition to this for the assessment of asymptotic stability of linear parametric circuit according to criterion 2 it will be enough: a) to find normal transfer function  $\mathbb{W}(s, \xi)$  of this circuit by frequency symbolic method in the form of truncated Fourier series (4) or (10); b) to find the roots of denominator  $\Delta(s)$  of the function  $\mathbb{W}(s, \xi)$ ; c) to determine the roots with null or positive real parts among roots of polynomial  $\Delta(s)$ . If such roots exist the circuit is unstable but if they do not exist the circuit is stable asymptotically.

### Example 1

Let's carry out the assessment of stability of single-circuit parametric amplifier shown in Figure 1.

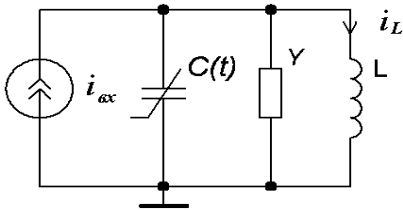


Fig. 1. Single-circuit parametric amplifier

$i_{ax} = Im \cdot \cos(\omega t + \varphi)$ ;  $Im = 1A$ ;  $\omega = 1rad/s$ ;  $Y = 0,25 S$ ;  $L = 1H$ ;  $C(t) = C_0 \cdot (1 + m \cdot \cos(\Omega t))$ ;  $C_0 = 1F$ ;  $\Omega = 2 \cdot \omega$

From current of signal  $i_{ax}(t)$  to current of inductor  $i_L(t)$  for the circuit from Figure 1 the normal parametric transfer function  $W(s, \xi)$  is determined by frequency symbolic method [3] in the form of truncated Fourier series, for example, when  $k=1$  it is:

$$\mathbb{W}(s, \xi) = \frac{A_0(s)}{\Delta(s)} + \frac{A_{c1}(s)}{\Delta(s)} \cdot \cos(i\Omega \xi) + \frac{A_{s1}(s)}{\Delta(s)} \cdot \sin(i\Omega \xi), \quad (14)$$

where

$$A_0(s) = A_0(s, m) = -0.0625 \cdot (4 \cdot s^3 + s^2 + 20 \cdot s + 4) \cdot s \cdot m;$$

$$A_{c1}(s) = A_{c1}(s, m) = 2.3125 + 0.625 \cdot s + 2.515625 \cdot s^2 + 0.125 \cdot s^3 + 0.25 \cdot s^4;$$

$$A_{s1}(s) = A_{s1}(s, m) = 0.5 \cdot s \cdot m \cdot (s^2 + 3);$$

$$\Delta(s) = \Delta(s, m) = 2.3125 + (0.25 - 0.125 \cdot m^2) \cdot s^6 + (0.1875 - 0.03125 \cdot m^2) \cdot s^5 + (2.796875 - 1.125 \cdot m^2) \cdot s^4 + (1.378906 - 0.125 \cdot m^2) \cdot s^3 + (4.984375 - 1.5 \cdot m^2) \cdot s^2 + 1.203125 \cdot s,$$

$m$  - intensity of modulation of capacity  $C(t)$ .

Expression (14) is solution to the following differential equation [1]:

$$\left[ 1 + (-LC'(\xi) + LY)s + LC(\xi)s^2 \right] \cdot W(s, \xi) + (LC'(\xi) - LY - 2LC(\xi)s) \cdot W'(s, \xi) + LC(\xi) \cdot W''(s, \xi) = 1 \quad (15)$$

which by-turn [1] issues from differential equation that describes circuit from Figure 1:

$$c(t)L \cdot i_L''(t) + [c'(t)L + yL] \cdot i_L'(t) + i_L(t) = i_{ax}(t). \quad (16)$$

Figure 2 shows trajectories of roots in the plane  $\sigma j\omega$  that are received during the change of  $m$  from 0,15 to 0,7 and with six harmonics in approximation  $\mathbb{W}(s, \xi)$ .

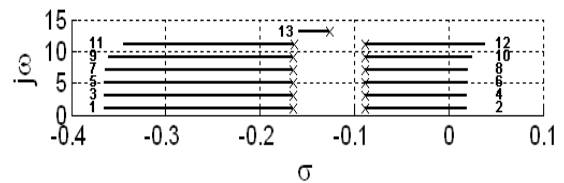


Fig. 2. Trajectories of roots of denominator  $\Delta(s, m)$  of function  $\mathbb{W}(s, \xi)$  of the circuit in Figure 1, during the change of  $m$  from 0,15 to 0,7 and for six harmonics in its approximation.

The beginning of every trajectory is signed by symbol «x» and at the end of trajectory there is root number which makes it.

Complex conjugate root corresponds to every root from Figure 2. Trajectories of these conjugate roots are symmetric relatively to the axis  $\sigma$  and they are not shown in the Figure 2.

The fragment of trajectory of the root 2 for value  $m$  from 0,544 to 0,564 across 0,004 is shown in Figure 3

on an enlarged scale and as it can be seen from the figure it crosses axis  $j\omega$  when  $m = 0,544 \pm 0,002$ .

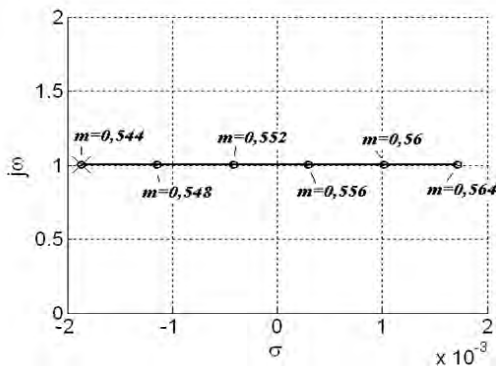


Fig. 3. Trajectory of root 2 during the change of  $m$  from 0,544 to 0,564 with lead 0,004. The beginning of trajectory is signed by symbol «x».

This means that at  $m < (0,544 \pm 0,002)$  the circuit from Figure 1 is stable asymptotically, but at  $m \geq (0,544 \pm 0,002)$  it is unstable. This result coincides absolutely with the result from the Figure 4 that is received for the circuit from Figure 1 by numerical method with the help of MicroCAP program which analyses electric circuits.

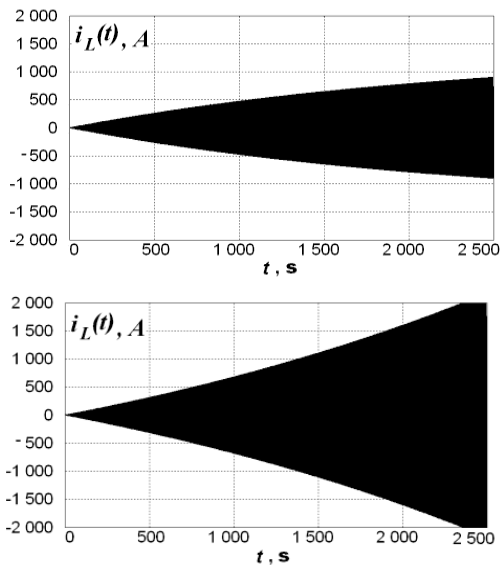


Fig. 4. Time dependences of current of inductor  $i_L(t)$  for the circuit from Figure 1 received with the help of MicroCap: a -  $m = 0,542$ , stable circuit; b -  $m = 0,546$ , unstable circuit

**Example 2**

To carry out the assessment of stability of double-circuit parametric amplifier shown in Figure 5. Signal circuit  $L1C1$  including its shunting elements  $C0, L2, C2$  is prepared for the frequency  $\omega_0 = 2\pi \cdot 10^8$  rad/s. The frequency of input signal is  $\omega_{c0} = \omega_0$  and the frequency of pumping is  $\Omega_0 = 2\pi \cdot 298,573$  rad/s.

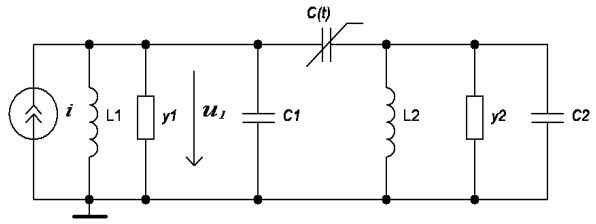


Fig. 5. Double-circuit parametric amplifier

$i(t) = Im \cdot \cos(\omega \cdot t + \varphi); \varphi = 45^\circ; Im = 0,1 \text{ mA};$   
 $y1 = y2 = 10^{-4} \text{ S}; C1 = C2 = 68 \text{ pF}; L1 = 36,70795 \text{ nH};$   
 $L2 = 9,312609 \text{ nH}; C(t) = C_0(1 + m \cdot \cos(\Omega \cdot t)); C_0 = 1 \text{ pF};$

Figure 6 shows the given trajectories of roots in plane  $\sigma j\omega$  that are received during the change of  $m$  from 0,16 to 0,28 at approximation  $\mathbb{W}(s, \xi)$  by two harmonics. The beginning of every trajectory in Figure 6 is signed by symbol “x”.

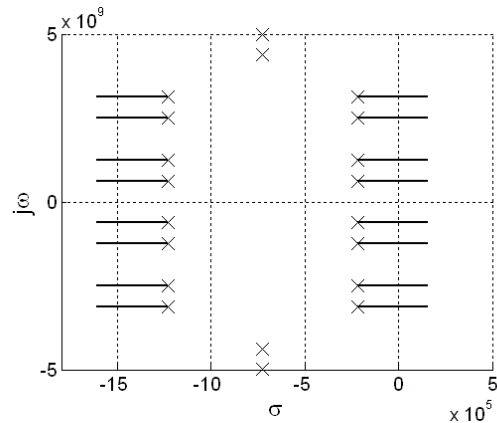


Fig. 6. Trajectories of roots of denominator  $\Delta(s, m)$  of function  $\mathbb{W}(s, \xi)$  of the circuit from Figure 5 during the change of  $m$  from 0,16 to 0,28 for two harmonics of its approximation.

The trajectories of the roots that cross axis  $j\omega$  for the values  $m$  from 0,215 to 0,245 with lead 0,01 (the position of the root on every lead is signed by the symbol “o”) are presented in Figure 7. As it can be seen from the figure the trajectories cross the axis  $j\omega$  when  $m = 0,23 \pm 0,005$ .

This means that at  $m < (0,23 \pm 0,005)$  the circuit from Figure 5 is asymptotically stable but at  $m \geq (0,23 \pm 0,005)$  it is unstable. This result coincides with the result received for the circuit from Figure 5 with the help of MicroCAP program.

**4. Conclusions**

From the material presented in the article the following conclusion can be drawn.

1. Unlike the assessment of stability of the interval, the assessment of asymptotic stability can be performed according to frequency symbolic method of analysis of linear parametric circuits during approximation of

normal function of circuit transfer performed by truncated Fourier series.

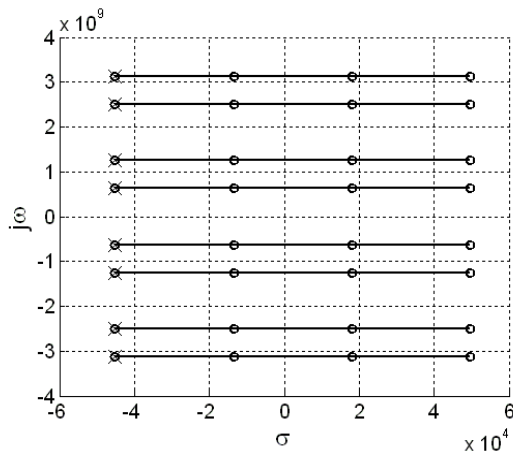


Fig. 7. Trajectories of roots during the change of  $m$  from 0,215 to 0,245 with lead 0,01. The beginning of trajectory is signed by symbol "x"

2. When using the frequency symbolic method of analysis the approximation  $W^{\epsilon}(s, \xi)$  in the form of (4) leads the problem of assessment of asymptotic stability of circuit from the analysis of convergence characteristic  $\rho = \chi(\sigma)$  of function  $W(s, r)$  to ordinary finding of denominator root  $\Delta(s)$  of a normal function of circuit transfer  $W(s, \xi)$  with the biggest real part.

3. The results of experiments in assessment of asymptotic stability carried out on the circuit in Figure 1 and the circuit in Figure 5 on the basis of the method offered in the article and MicroCAP program coincide and it proves the correctness of the delivered material.



**Yuriy Shapovalov**

PhD, associate professor of the Department of Radioelectronic Devices and Systems, Lviv Polytechnic National University. The research area is frequency symbolic analysis of linear parametric circuits with constant and variable parameters.



DSc, professor of the Department of Theoretical Radio-Engineering and Radio-Measuring, Lviv Polytechnic National University. His research and teaching experience makes up over 40 years. He is the author of over 250 scientific papers and 2 monographs. He has prepared 9 candidates (PhD) and 3 doctors of technical sciences.

The research area is analysis of non-linear electronic circuits and methods of reliability control of radio electronic devices and communication equipmen.

## References

1. Солодов А.В., Петров Ф.С. Линейные автоматические системы с переменными параметрами. - М.:Наука, 1971.-620 с.
2. Бриккер И.Н. О частотном анализе линейных систем с переменными параметрами//Автоматика и телемеханика, № 8, 1966.-с.43-54.
3. Шаповалов Ю., Мандзій Б. Символьний аналіз лінійних параметричних кіл: стан питань, зміст і напрямки застосування. // Теоретична електротехніка. 2007. Вип. 59 с.3-9.
4. Шаповалов Ю.І. Особливості оцінки асимптотичної стійкості лінійних параметричних кіл частотним символьним методом. // Моделювання та інформаційні технології. Зб. наук. пр. ПІМЕ НАН України. – Вип.55. – К.: 2010. – с. 126-133.
5. Шаповалов Ю.І., Маньковський С.В. Застосування топологічних методів за символьним аналізу лінійних параметричних кіл. Вісник НУ «Львівська політехніка» Радіоелектроніка та телекомунікації, № 618, 2008.-с. 76-81.
6. Zadeh L. A., "Frequency Analysis of Variable Networks," Proc. of the IRE, vol.39,1950.

## ТЕОРЕТИЧНІ ОСНОВИ ОЦІНКИ АСИМПТОТИЧНОЇ СТІЙКОСТІ ЛІНІЙНИХ ПАРАМЕТРИЧНИХ КІЛ ЗА ЧАСТОТНИМ МЕТОДОМ

Ю. Шаповалов, Б. Мандзій

Показана можливість оцінки асимптотичної стійкості лінійних параметричних кіл за частотним символьним методом та апроксимацією передавальної функції кола зрізаним рядом Фур'є. Оцінка асимптотичної стійкості здійснена за допомогою нормальної передавальної функції, а не бічастотної, як це робиться зазвичай. Наведено два приклади оцінки асимптотичної стійкості параметричних підсилювачів.