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# APPROXIMATE MATHEMATICAL MODELS OF ELECTROMAGNETIC AND THERMAL PROCESSES AT INDUCTION HEATING OF METAL STRIPS

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**Abstract**. Electromagnetic and thermal processes in a moving conducting strip have been considered on the base of a simplified mathematical model. The following features have been taken into account: non-uniformity of eddy current and Joule's heat distributions, heat transfer in directions across the strip and along its surface. The temperature has proved to become homogeneous through-thickness for typical modes of induction heating. On the contrary, the heat transfer along the surface is insignificant and, therefore, it is possible to consider the process adiabatic. Estimations of characteristic parameters of the process have been made for aluminum, brass and steel strips.

**Key words** – high-frequency induction heating, metal strip, mathematical models, electromagnetic and heat-transfer parameters

## 1. Introduction

One of the main problems of induction heating of metal strip is to ensure a certain temperature of the strip moving across an alternating electromagnetic field of an inductor [1]. In general, the problem is complicated in the computation sense. Therefore, the approximate asymptotic methods of calculation which allow us to consider the most essential geometrical, electrophysical and heat-transfer properties of electromagnetic systems are rational.

The main objective is the analysis of electromagnetic and heat-transfer parameters of an electromagnetic system of high-frequency induction heating of metal strips to define the possibility of using approximate mathematical models and to determine conditions under which it is possible to consider electromagnetic and thermal problems separately.

In the present paper the high-frequency induction heating of strips is considered. The field is generated by a coreless inductor, made in the form of a coil frame generally of a spatial configuration (fig. 1) [2, 3].

Earlier in [2, 4] with the use of the method of asymptotic expansion for a field created by a current contour located above the conducting half-space, the analytical estimations of geometrical parameters of the system were determined under condition of a uniform distribution of linear density thermal energy generated in the strip that moves in the field of inductor. This distribution of thermal energy defines the distribution of temperature of strip. The problem will be simplified if the heat transfer along a surface of the strip is insignificant.

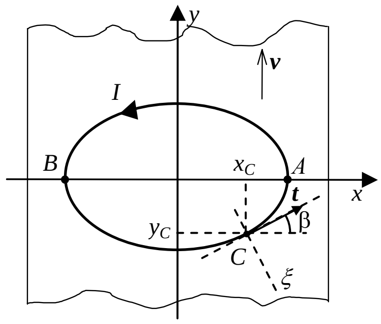


*Fig. 1. The model of electromagnetic system*

## 2. Mathematical model

The field is considered to be generated by a coreless inductor, made in the form of a coil frame. The temperature of the strip is relatively low that alows heat radiation not to be taken into account.

At high-frequency induction heating of metal strips, the following three dimensionless parameters turn out to be small:

$$\varepsilon_1 = \frac{\delta}{d} = \frac{1}{d}\sqrt{\frac{2}{\omega\mu\mu_0\gamma}}\,, \qquad (1)$$

$$\varepsilon_2 = \frac{1}{h}\left(\frac{\mu}{\omega\mu_0\gamma}\right)^{1/2}, \qquad (2)$$

$$\varepsilon_3 = \frac{v}{\omega h}\,, \qquad (3)$$

where $\delta$ is penetration of field, $\omega$ is frequency, $\gamma$ is electro conductivity, $\mu$ is relative magnetic conductivity, $d$ is thickness of strip, $h$ is distance from contour to strip, $v$ is velocity of strip.

In the devices intended for induction heating the inductor is located near to the surface of the strip. In this case one more parameter is usually small

$$\varepsilon_4 = h/D \tag{4}$$

where $D$ is the representative size of the inductor contour.

If the above introduced parameters are small the electromagnetic problem on distribution of eddy currents in conductive medium becomes considerably simpler. In [4] it has been shown that the average for a period density of the electromagnetic energy flow along metal surface $p_z$ (the real part of normal component of complex Poynting vector)

$$p_z = \mathrm{Re}(-\mathbf{\Pi} \cdot \boldsymbol{e}_z) \tag{5}$$

can be presented in the form of bounded numbers of asymptotical series. To conduct estimation in case of a high frequency field it is enough to take the first nonzero item:

$$p_z = \frac{I^2\zeta}{\pi^2 h^2} \cdot \frac{1}{\left(1+\xi^2/h^2\right)^2}, \tag{6}$$

where $\zeta = \sqrt{\dfrac{\omega\mu\mu_0}{2\gamma}}$ - the absolute value of surface impedance.

As (6) shows the thermal energy is produced in a narrow area of an $h$ width strip under the conductor contour. Transfer of the generated heat occurs in directions across strip and along its surface in two directions: parallel and perpendicularly to velocity $v$.

All medium physical properties: conductance, thermal conductivity, a specific heat – are considered as the linear for heat parameters estimation.

## 3. Heating parameters estimation

### Temperature distribution along thickness

The representative size in a transversal direction to the strip is penetration of an electromagnetic field into its metal $\delta$. For high-frequency heating the depth $\delta$ is usually considerably less than the strip thickness. Due to the thermal conductivity the extracted heat penetrates deeper into the metal. If this process is sufficiently quick, at a certain time the uniform temperature will be established along strip thickness. Let's compare the representative temperature stabilisation time for the thickness $\tau_d$ to the representative time of strip heating $\tau_p$.

As in actual practice the thickness of strip $d$ is much less than distance $h$ the temperature at any point of the medium can be determined from expression:

$$T(z,t) = \frac{Q}{c\rho\sqrt{\pi a t}} e^{-\frac{z^2}{4at}}, \quad z \le 0, \tag{7}$$

where $Q$ is surface density heat, thermal diffusivity $a = \lambda / c\rho$ determined through specific heat capacity $c$, thermal conductivity $\lambda$ and density $\rho$, $t$ is time. If in a certain time the temperature of the given volume of a metal strip becomes homogeneous along thickness, in the absence of a heat emission it will be equal to $T_\infty = \dfrac{Q}{c\rho\, d}$.

The estimation of stabilisation time of the homogeneous temperature $\tau_d$ we will calculate from the following condition: $T(0,\tau_d) = T_\infty$. From here the estimation of the temperature stabilisation time is $\tau_d \sim \dfrac{c\rho\, d^2}{\pi\lambda}$.

The representative time of strip heating $\tau_p$ can be determined as time of the metal strip section transit under conductor. The following parameter allows to come to a conclusion, whether the homogeneous temperature in the process of the heating is established:

$$\varepsilon_d = \frac{\tau_d}{\tau_p} = \begin{cases} \dfrac{c\rho\, d^2 v\cos\beta}{\pi\lambda h}, & \text{far from edges (point } C\text{)} \\ \dfrac{c\rho\, d^2 v}{\pi\lambda D}, & \text{near to points } A \text{ and } B. \end{cases} \tag{8}$$

Table 1 contains values $\varepsilon_d$ for strips with thicknesses of $d = 10^{-3}$ m and $d = 3\cdot 10^{-3}$ m (the line separates values corresponding to the upper and lower expressions in (8) correspondingly). The following values of conductor dimensions and velocity are typical for the induction heating: $D = 0{,}2$ m, $h = 3\cdot 10^{-2}$ m, $\cos\beta = 1$, $v = 10^{-1}$ m/s. Parameters $\varepsilon_d$ are given for the following materials: aluminium ($c = 8{,}8\cdot 10^2$ J/kg·K, $\rho = 2{,}7\cdot 10^3$ Kg/m$^3$, $\lambda = 2{,}1\cdot 10^2$ W/m·K); brass ($c = 3{,}8\cdot 10^2$ J/kg·K, $\rho = 8{,}5\cdot 10^3$ Kg/m$^3$, $\lambda = 85{,}5$ W/m·K); steel ($c = 4{,}6\cdot 10^2$ J/kg·K, $\rho = 7{,}8\cdot 10^3$ Kg/m$^3$, $\lambda = 45{,}4$ W/m·K).

*Table 1*

**Parameter** $\varepsilon_d$

| Thickness $d$, m | Aluminium | Brass | Steel |
|---|---|---|---|
| $10^{-3}$ | $1{,}2\cdot 10^{-2}$ / $1{,}8\cdot 10^{-3}$ | $4\cdot 10^{-2}$ / $6\cdot 10^{-3}$ | $8{,}4\cdot 10^{-2}$ / $2\cdot 10^{-3}$ |
| $3\cdot 10^{-3}$ | $0{,}11$ / $1{,}6\cdot 10^{-2}$ | $0{,}36$ / $5{,}5\cdot 10^{-2}$ | $0{,}75$ / $1{,}1\cdot 10^{-1}$ |

Apparently that practically always the temperature becomes thickness homogeneous already during the strip transit under the corresponding sections of a contour. Therefore, in mathematical models of induction heating the temperature of metal strips can be accepted everywhere homogeneous for a thickness except for the sections transiting at present time under a contour with a current.

**Heat transfer along a metal strip surface**

The extracted heat energy is transferred in the course of the motion in the direction of the velocity of the strip transit. On the other hand, the strip temperature directly under the contour strongly increases, and because of the emerged temperature gradient there is the heat flow caused by thermal conductivity. In this case the problem consists in the comparison of two processes of the heat transfer: thermal conductivity and the transit of heat by motion of metal. Parameters of processes of the heat transfer are different for the sections near to the intermediate point $C$ and at edges of the contour near to the points A and B (fig. 1). Therefore, we will consider these sections separately.

a) Heat transfer near to the intermediate points of contour.

During transition under contour, the element of volume of the strip $\Delta V = \Delta x \Delta y d$ which have reached the coordinate $y$ receives energy, equal to [4]

$$\Delta W(y) = \frac{\Delta x \Delta y}{v} \int_{-\infty}^{y} p_z(\xi, h)\, d\xi \qquad (9)$$

This thermal energy leads to temperature growth:

$$\Delta W(y) = c\rho \Delta x \Delta y d (T(y) - T_0) \qquad (10)$$

where $T_0$ - temperature before heating. Comparing (9) and (10), we obtain

$$T(y) - T_0 = \frac{1}{c\rho\, dv} \int_{-\infty}^{y} p_z(\xi, h)\, d\xi \qquad .(11)$$

The temperature gradient and medium motion cause the corresponding thermal flows. The Fig. 2 qualitatively illustrates directions of flows of thermal energy related to thermal conductivity $U_1$ and heat transfer by a medium motion $U_{v1}$.

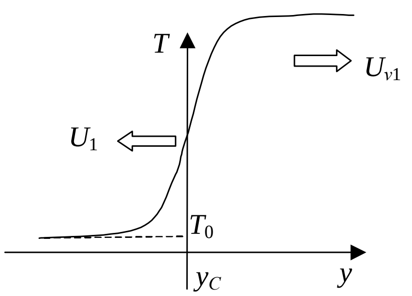


*Fig. 2. The directions of heat flows under intermediate points of contour*

The heat flow due to the thermal conductivity is directed in opposite direction to strip motion, and is equal to

$$U_1(\xi) = -\lambda \frac{dT}{dy} \Delta x d = -\frac{\lambda \Delta x}{c\rho v} p_z(\xi, h) \qquad (12)$$

The maximum value $U_{1\max}$ is reached directly under a contour at $\xi = 0$.

Let's compare $U_{1\max}$ to a maximum of heat flow transferred by a motion of heating metal $U_{v1} = c\rho\, v \Delta x d (T(\infty) - T_0)$. In order to describe the heat transfer by the thermal conductivity in the direction along the velocity vector in comparison with a heat transfer by the motion let's introduce the following parameter:

$$\varepsilon_{L1} = \left| \frac{U_{1\max}}{U_{v1}} \right| = \frac{\lambda}{c\rho\, v} \cdot \frac{p_z(0, h)}{\int_{-\infty}^{\infty} p_z(\xi, h)\, d\xi} \qquad (13)$$

Considering (6), the resultant expression for $\varepsilon_{L1}$ is given by:

$$\varepsilon_{L1} = \frac{2\lambda}{\pi\, c\rho\, h\, v} \qquad (14)$$

Table 2 contains values of parameter $\varepsilon_{L1}$ for aluminium, brass and a steel strips at $h = 3 \cdot 10^{-2}$ m and motion velocity $v = 10^{-1}$ m/s.

*Table 2*

**Parameters $\varepsilon_{L1}$ and $\varepsilon_{L2}$**

| | Aluminium | Brass | Steel |
|---|---|---|---|
| $\varepsilon_{L1}$ | $1{,}9 \cdot 10^{-2}$ | $5{,}6 \cdot 10^{-3}$ | $2{,}7 \cdot 10^{-3}$ |
| $\varepsilon_{L2}$ | $2{,}6 \cdot 10^{-1}$ | $7{,}6 \cdot 10^{-2}$ | $3{,}6 \cdot 10^{-2}$ |

In all cases and even at a velocity in some cm/s heat transfer gradient of temperature in the area of the conductor of the contour the thermal conductivity is significantly lesser in comparison with a heat transfer due to the metal motion.

b) Heat transfer near contour edges.

Near the edges of the current contour the temperature gradient is directed perpendicularly to the velocity $v$. Heat input happens on the long section. To get the estimates, we will consider that heat is brought uniformly throughout a section, equal to the representative size of contour $D$.

The amount of heat brought to the strip section of a small length $\Delta y$ and a width of $2\xi$ will be

$$\Delta W_{v2} = \int_{-\xi}^{\xi} p_z(\xi, h) \Delta y \Delta\, t\, d\xi \qquad (15)$$

where $\Delta\, t = D / v$ – section heating time.

Quantity of heat, given away by thermal conductivity is

$$\Delta W_2 = -\frac{\lambda d \Delta y}{v} \int_{-D/2}^{D/2} \frac{\partial T(\xi, y)}{\partial \xi} dy \qquad (16)$$

The direction of thermal energy flow is shown on Fig. 3.

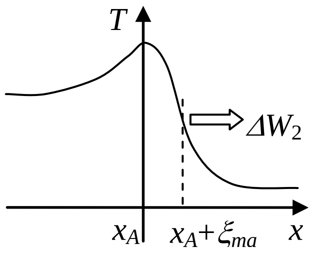


*Fig. 3. The direction of heat flow near contour edges*

The temperature linearly grows in process of motion along coordinate $y$ :

$$T(\xi, y) - T_0 = \frac{p_z(\xi, h)}{c\rho\, dv} y, \qquad \xi \geq 0 \tag{17}$$

Therefore we get

$$\Delta W_2 = \frac{\lambda}{c\rho} \cdot \frac{\Delta y D^2}{2v^2} \cdot \frac{\partial\, (p_z(\xi, h))}{\partial \xi} \tag{18}$$

The ratio of heat quantity $\Delta W_2$ leaved the strip element with width of $2\xi$ due to thermal conductivity to arrived energy $\Delta W_{v2}$ in metal strip caused by eddy currents is

$$\frac{\Delta W_2}{\Delta W_{v2}} = \frac{D}{2v} \cdot \frac{\lambda}{c\rho} \cdot \frac{\dfrac{\partial\, (p_z(\xi, h))}{\partial \xi}}{\int\limits_{-\xi}^{\xi} p_z(\xi, h) d\xi} \tag{19}$$

Taking into account (6) the factor in (19) becomes

$$F(\chi) = \frac{\partial\, (p_z(\xi, h))}{\partial \xi} \Bigg/ \int\limits_{-\xi}^{\xi} p_z(\xi, h) d\xi = = \frac{16\chi}{h^2 (1+\chi^2)^2 \cdot \left[\chi + \left(1+\chi^2\right) arctg(\chi)\right]} \tag{20}$$

where $\chi = \xi / h$ .

The greatest magnitude $\Delta W_{2\max}$ will be at $\chi = \chi_{\max}$ , where the derivative $\frac{\partial\, (p_z(\xi, h))}{\partial \xi}$ has maximum value. In this point $\chi_{\max} = 1/\sqrt{5}$ and then $F(\chi_{\max}) = 2{,}6$ .

The dimensionless parameter is equal:

$$\varepsilon_{L2} = \frac{W_{2\max}}{W_{v2}} = 1{,}3 \cdot \frac{\lambda}{c\rho} \cdot \frac{D}{vh^2} \tag{21}$$

In Table 2 values of $\varepsilon_{L2}$ for the former parameters of heating are presented. As it seen in most cases the heat transfer due to the thermal conductivity near the edges remains negligible in comparison with heat income, caused by Joule dissipation of an electromagnetic energy. Only at rather small velocities thermal flows can appear comparable, especially for materials with high value of thermal diffusivity (aluminium, cuprum, etc).

**4. Conclusions**

As a result of the fulfilled estimation of main parameters of heating moving metal strips in a high-frequency field of the ironless conductors, the following conclusions, which can be used for the elaboration of mathematical models of the considered processes, have been drown:

1) The temperature can be considered thickness homogeneous at any point of the metal strips out of the area directly under a current contour.

2) In directions along the surface of the heated metal strip the heat transfer due to the thermal conductivity is negligibly small in comparison with the heat transfer, caused by the medium motion and the heat income due to the Joule dissipation of electromagnetic energy. In this sense it is possible to assume that it is an adiabatic process in these directions. The temperature at any point of moving strip (with the restrictions introduced in the previous section) will be:

$$T(x) = \frac{P(x, y)}{c\rho v d} \tag{22}$$

where $P(x, y) = \int\limits_{-\infty}^{y} p_z(\xi, h) d\xi$ is linear density of electromagnetic energy flow caused by Joule dissipation.

The above conclusions are illustrated below by comparison of the calculations results for the brass strip temperature, performed with assumption of adiabatic heating and taking into account the thermal conductivity. The strip with thickness of $d = 3 \cdot 10^{-3}$ m and width of $0{,}6$ m is heated up, transiting under a flat circular contour with radius of $a = 0{,}25$ m. Distance between contour and the brass strip is $h = 0{,}04$ m. The frequency of current is $f = 10^4$ Hz. Calculations have carried out for two strip's velocity values: $v = 0{,}25$ m/s and $v = 0{,}01$ m/s.

With the chosen input data, parameters $\varepsilon_{L1}$ and $\varepsilon_{L2}$ have the values shown in table 3.

*Table 3*

**Parameters $\varepsilon_{L1}$ and $\varepsilon_{L2}$ for brass**

| $v$ , m/s | $\varepsilon_{L_1}$ | $\varepsilon_{L_2}$ |
|---|---|---|
| 0,25 | $1{,}7 \cdot 10^{-3}$ | $4{,}3 \cdot 10^{-2}$ |
| 0,01 | $4 \cdot 10^{-2}$ | 1,1 |

Table 3 shows that along a surface of a strip for a velocity of $v = 0{,}25$ m/s the process is adiabatic. At the same time for a strip velocity of $v = 0{,}01$ m/s it is impossible to consider the process as adiabatic and for the determination of distribution temperature it is necessary to consider thermal conductivity of a strip.

The above conclusions proved to be true by the calculations of the distribution temperature on a strip width on distance of 0,5 m from the centre of the circular contour. In Fig. 4 and Fig. 5 the solid curves represent the results obtained with the application of asymptotic computational method [2] with assumption of adiabatic heating. The dotted curves correspond to the data of joint solution of the electromagnetic and thermal problems obtained by method in which no restrictions are put on the parameters of heating [3] (the data were submitted by Dr. I.P.Kondratenko).

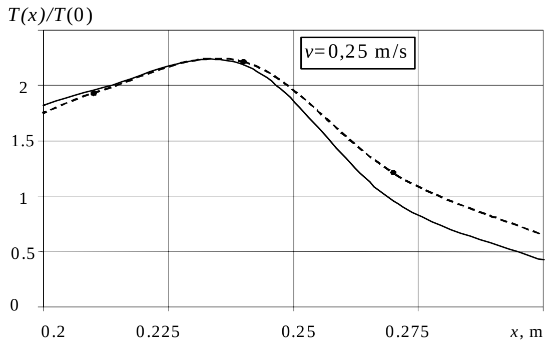


*Fig. 4. Distribution of temperature at v=0,25 m/s*

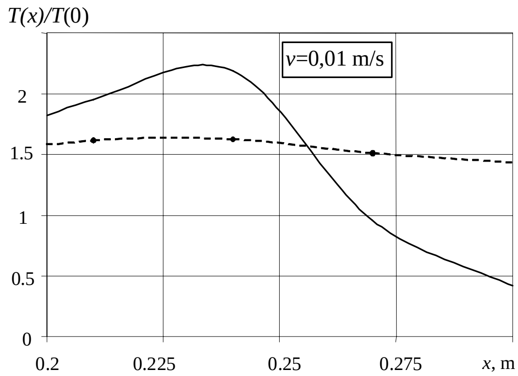


*Fig. 5. Distribution of temperature at v=0,01 m/s*

The comparison shows that at the strip velocity of $v = 0{,}25$ m/s there is a good match of the results of two approaches and in this case the condition of adiabatic heating is really satisfied.

For a strip velocity of $v = 0{,}01$ m/s as it follows both from estimates of parameters and from the results of the calculations (fig. 5), it is impossible to consider the heating as adiabatic process. The calculations in this case should be performed taking into account the joint influence of the heating by eddy currents and the heat transfer related to thermal conductivity and the motion of the medium.

**References**

1. Rudnev V., Cook R., Loveless D., Black M. Induction heat treatment.– Marcel Dekker Inc., 1997.– 872 p.

2. Vasetsky Y., Mazurenko I. Electromagnetic systems geometrics for high-frequency induction heating of metal strips // Tekhnichna elektrodynamika. – 2009.– № 5.– p. 9-15.

3. Vishtak T., Kondratenko I., Rashepkin A. Band induction heating by current contours of a canonical form // Tekhnichna elektrodynamika.- 2003.- № 1.- p. 63-68.

4. Vasetsky Y., Mazurenko I. Parameters of electromagnetic systems with eddy currents for using in approximate mathematical models // Proceedings of the VII Intern. Workshop: Computation Problems of Electrical Engineering. Odessa, August 27-30, 2006. – P. 190-193.

## НАБЛИЖЕНІ МАТЕМАТИЧНІ МОДЕЛІ ЕЛЕКТРОМАГНІТНИХ І ТЕПЛОВИХ ПРОЦЕСІВ ПРИ ІНДУКЦІЙНОМУ НАГРІВАННІ МЕТАЛЕВИХ СМУГ

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На основі спрощеної математичної моделі розглянуто електромагнітні та теплові процеси в рухомій електропровідній смузі. При індукційному нагріванні тепло перенос теплопровідністю уздовж поверхні смуги виявляється незначним, і процеси у цьому напрямку можна розглядати як адіабатичні. Для металевих смуг з алюмінію, латуні і сталі зроблено оцінки характерних параметрів процесів.

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