COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 1, No. 1, 2011

TRAFFIC ROUTING IN TELECOMMUNICATION NETS AND ITS DIAKOPTICS REPRESENTATION

M. Klymash, B. Strykhalyuk, M. Kaidan, I. Demydov

Lviv Polytechnic National University, 12, St. Bandera Str., Lviv, 79013, UA

mklimash@polynet.lviv.ua

Abstract. Characteristics of flow-oriented models of routing have been classified. The model analysis has carried out for routing, that is accepted as a basis for the existing telecommunication network protocols. The decomposition representation of telecommunication networks has been performed and a calculation algorithm by diakoptics has been proposed.

Keywords: diakoptics, tensor, routing, telecommunication networks

1. Introduction

Despite widely escalated graph (graph-combinatory) models of routing [1, 2], that is accepted as a basis for most of the existing routing protocols - RIP (Routing Internet Protocol), IGRP (Interior Gateway Routing Protocol), EIGRP (Extended IGRP), IS-IS (Intermediate System - to - Intermediate System), OSPF (Open Shortest Path First), PNNI (Private Network - to - Network Interface), at present just flow-oriented models of routing are ever-broadening requested. On the one hand they consider the dataflow character of present-day traffic as mostly multimedia-based (voice, video, etc.), but on the other hand they are more adopted for load-balancing solutions and quality of service provision in multi-service NGN telecommunications.

An analysis of recent investigations and publications carries inference about the fact that a lot of approaches to float modeling for routing have been proposed, in the frame of the existent and future telecommunications technologies [3-6]. Depending on features consideration, deepness of structural, functional architecture and operation of telecommunication network, we obtain an optimization task using an appropriate mathematical model. Generally, there are a few methods to calculate a necessary path (set of paths) [7]. To solve those optimization tasks, a set of different methods should be used, combinatory Dijkstra algorithms (OSPF, IS-IS, PNNI), Bellman-Ford (RIP, IGRP, EGRP), Floyd-Warshall [3, 6] for graph models, but for flow-models methods of the mathematical programming and optimal control are more suitable [4,5].

For each of the flow models, their related algorithms and methods to solve routing tasks there are specific features, conditions and an implementation branch. Unfortunately, in the early-known works dedicated to the analysis of different routing models rather general conclusions were made [4, 6, 8, 9, 16, 17]. It is difficult, and sometimes impossible to effectively determine their benefits, shortcomings and most optimal conditions to use [10].

The aim of this work is to obtain numeric results of the comparative flow models analysis of different network topologies and users' network traffic characteristics. The comparative analysis of flow routing models in the terms of load-balancing and providing service quality allows to determine their further application.

2. The classification of flow routing models

The comparative analysis was carried out for five basic flow routing models [11]:

M-1. The one-path routing model, based on searching the shortest path which consists of the minimal number of hops as in RIP v1 [12].

M-2. The multi-path routing model (MPRM), unlike the previous M-1, supports load balancing through paths that is equivalent by the cost (length). This provides solutions to the routing tasks by linear programming through minimization of the objective function

$$Y = c\,x, \qquad (1)$$

where c – cost (metric) coefficients vector by dimension n, all coordinates of which are set to 1, in other words $c_{i,j}=1$ ($i,j=\overline{1,m}$ 1,m; $i \neq j$); n – the number of transmission lines; m – the number of network nodes telecommunication network; x is the required vector with coordinate $x_{i,j}$ that simulates the traffic intensity (1/c) in the transmission line (i, j).

According to the physical aspect of the tasks under solution x-vector coordinates are merged by a set of constraints which simulate conditions of the flow conservation into each network node [4]:

$$\begin{cases} \sum x_{i,j} - \sum x_{j,i} = 0 & \textit{– for the transit node}; \\ \sum x_{i,j} - \sum x_{j,i} = r_{in} & \textit{– for the initiate node}; \\ \sum x_{i,j} - \sum x_{j,i} = -r_{in} & \textit{– for the recipient node}, \end{cases} \qquad (2)$$

where r_{in} – the entering network traffic intensity.

M-3 is the MPRM model with introduced metric of IGRP with load balancing through paths that is not-equivalent by the cost (length). M-3 is represented by (1)-(2). Unlike M-2, M-3 has IGRP metrics corresponding to c-vector coordinates (1), are values of the transmission line throughput [1]:

$$c_{i,j}= 10^7/\varphi_{i,j}, \qquad (i,j=\overline{1,m}\ 1,m\ ;\ i \neq j). \tag{3}$$

To formalize conditions of transmission lines overload prevention with (2) additional constraints are introduced:

$$x_{i,j} \le \varphi_{i,j}, \qquad (i,j=\overline{1,m}\ 1,m;\ i \neq j). \tag{4}$$

where $\varphi_{i,j}$ – transmission line throughput (1/c).

M-4. The MPRM model, proposed by Gallager [3] which solves the routing task by non-linear programming with maximizing the objective function:

$$Y = \max_{(i,j)} \left\{ \frac{x_{ij}}{\varphi_{ij}} \right\}. \tag{5}$$

Definition (5) provides the transmission lines utility coefficient mini-max optimization. Thus, we must provide the following flow conservation conditions:

$$\gamma_{i,j} = r_{i,j} + \sum_{k \in M^i} \gamma_{k,j} \phi^k_{j,i}, \quad \text{at}$$

$$\sum_{k \in M^i} \gamma_{k,j} \phi^k_{j,i} = x_{i,k}, \tag{6}$$

where M^i is the set of neighbor nodes to *i*; $r_{i,j}$ is the entering traffic intensity (1/c), which is received from users to the node *i* for node *j*; $\gamma_{i,j}$ is the sum of the entering flow fed onto the node *i* and that from neighbor nodes to *j*; $\phi^k_{j,i}$ – routing variable i.e. the part of the flow $\gamma_{i,j}$, that was transmitted by node i through line (i,k).

For the routing variables the following conditions are applied:

$$\varphi^k_{j,i} = \begin{cases} 0, & \text{if } i = j; \\ \ge 0, & \text{if } i \neq j, \end{cases} \qquad \sum_{k \in M^i} \phi^k_{j,i} = 1. \tag{7}$$

M-5. The Gallager's MPRM model (5)-(7) which was developed in [13] by introducing the provision conditions for QoS by packets transmissions speed, their average delay, jitter and in-time delivery probability. In consequence of limited transmission lines throughput in the model M-5, as in models M-3 and M-4 the limitation for (4) was applied.

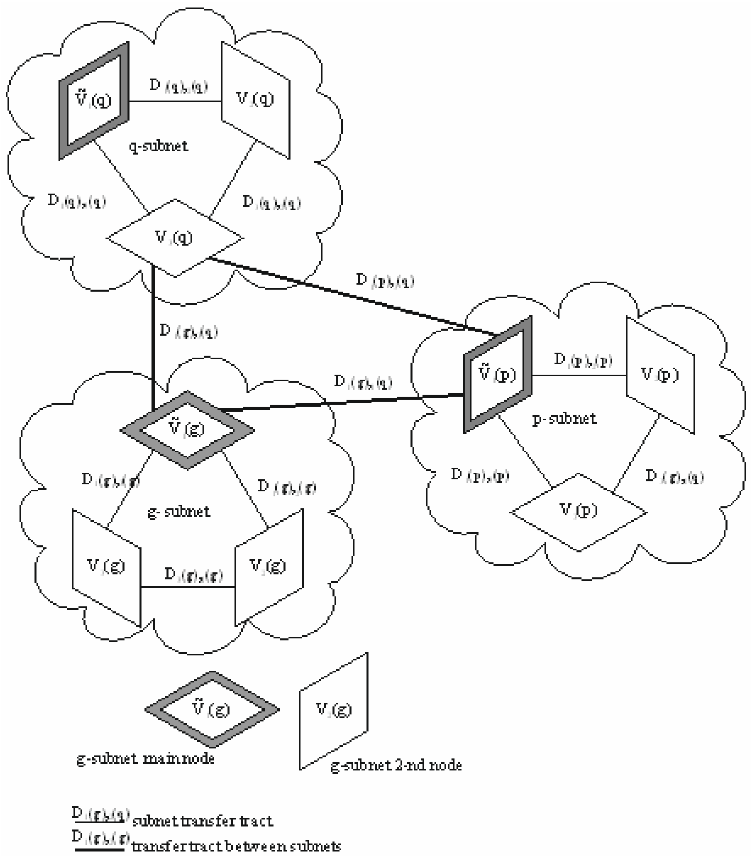

Fig.1. Generalized structure of the complex telecommunication network, presented as non-oriented graph

3. The decomposition representation of the telecommunication network using diakoptics

The telecommunication network structure is usually presented as a non-oriented graph (Fig.1). The set of the apices V of the given graph consists of nodes (routers) of the telecommunication network (V_j, j=$\overline{1,m}$), D is the set of the transmission lines between nodes ($D_{i,j}$; i, j=$\overline{1,m}$; i≠j). Let us use a hierarchical network representation as subnets which could be presented as $\mathbf{J}_1(V_{(q)},d_{(q)})$ of the graph $\mathbf{J}_2(V,d)$, where $V_{(q)}$ is the subset of the nodes which are the part of q-th subnet with number of the nodes - m_q. $D_{(q)}$ is the set of the transmission lines from incidental nodes of q -th subnet.

As you can see in Fig.1 the given network consists of a set of subnets, and one of the transition ways from a complex system to more simple is dividing into separate parts, i.e. decomposition. One of the most effective decomposition methods consisting in finding solution of some tasks for the systems is diakoptics [14].

The topology structure consists of subnets, in other words subsets $\mathbf{J}_1$, which are concatenated with each other by subset $\mathbf{J}_2$ [15]. The merged topological structure is the whole network and it is mathematically represented as **J**. Structural sets $\mathbf{J}_1$ and $\mathbf{J}_2$ are concatenated to structural set $\mathbf{J}=\mathbf{J}_1\times\mathbf{J}_2$.

To calculate the telecommunication network by diakoptics we set a system and write down its state equation in symbolic form, for example by Little's formula

$$\mathbf{H}=\mathbf{LT}, \quad (8)$$

where **H** is loading; **L** is a tensor that represents throughput; **T** is a time delay.

The task consists in solving the equation (8), where H and L are given and time delay is undefined:

$$\mathbf{T} = \mathbf{HL}^{-1}, \quad (9)$$

The existent tensor model of a telecommunication network which was founded by Pasechnikov [15] is based on the further generalization and representations by Little's equation in tensor form. The similar approach was stipulated for the similarity of (8) and Ohm law which was generalized and developed in the well known works of American scientist G. Kron [14, 16] who proposed tensor network analysis. But a telecommunication network is a more complex system than an electrical network. That's why being based on this analogy the tensor approach loses its adequacy while describing telecommunication network, in particular its structural and operational properties.

Therefore, it has been proposed to write down (9) as following:

$$T^{ij} = H_{kl}(L^{-1})^{ijkl},$$

where T^{ij} is the delay time between nodes *i* and *j*; H_{kl} is the loading traffic between nodes *k* and *l* at the network; $(L^{-1})^{ijkl} = F^{ijkl}$ is the tensor of inverted throughput in the network between nodes *i* and *j*, in case of the traffic transmitting between nodes *k* and *l*.

This generalization allows consideration of the multi-flow character of the network traffic, service differentiation for packets in the network nodes with necessity of the QoS provision by some qualitative parameters simultaneously.

At the initial stage we have set the investigated system, but solutions to equation (1) are absent.

The first stage. The model differentiation.

Let us to break the model into three separate parts of subnet $\mathbf{J}_1$. Thus, the cuts are broken by pairs of nodes, not contours. The separation of the parts must be absolute, so their interaction must be eliminated. It is important to avoid incoherent components between subsystems.

The second stage. The broken connections elimination.

All the cyclic interconnections have been eliminated from the investigated system. There is no intersystem connections kept in subsystems which could indicate how the subsystems were connected. The divided branches do not belong to none of the *n* subsystems of the initial system. As we cannot ignore them we must refer them to the additional *n+1*-th subsystem which we must construct.

The most evident benefit of the interconnection elimination between structures is that undefined delays for $\mathbf{J}_2$, will not be additional undefined variables for each subsystem $\mathbf{J}_1$. Thus, the number of undefined variables has essentially decreased.

The third stage– to obtain and solve equation (9) for each subsystem.

The fourth stage is dedicated to the solving of the equation system which is called crossing circuit.

When the initial system is divided into the ***n*** subsystems $\mathbf{J}_1$ (in our case according to Fig.1 the number of subsystem ***n***=3), we could consider that each of them with N loadings H after its solving forms N-dimensional abstract space (in our case according to Fig.1. N=3 – the number of nodes of subsystem $\mathbf{J}_1$). These N-dimensional spaces in the initial interconnected system will not be independent on each other and will be crossed or concatenated.

As (*n+1*)-th cuts' system (in Fig.1 it is presented as structure $\mathbf{J}_2$) plays a central role in the decomposition method let us define four steps to form it:

I. Critical fragments are selected from the complete structures of subsystems only. On the basis of this fragments we form the inverted throughput matrix

$F^{1'}=L'$, which is the basis for pivotal crossing circuit determination.

Consequently defining $C_1^{(1)}$, $C_1^{(2)}$ and $C_1^{(3)}$ – matrices for conversion of F^1 to $F^1_{(1)}$, $F^2_{(1)}$ and $F^3_{(1)}$ which refer to each subsystem $\mathbf{J}_{1,}$ respectively, we obtain following equation:

$$F^{1'} = C_{1t} F^1 C_1, \qquad (10)$$

II. The eliminated connections are recomposed without considering the throughput.

The recomposed transmission lines could be done with the help of the matrix C_2, which refers to the number of node's connections. This matrix allows re-composition of transmission lines as they were before elimination, but after decomposition.

III. Merging of subsystems fragments and basic cut system creation f'.

We should merge the subsystems with the help of the matrix C_3.

Given three converting matrices, we should obtain the matrix for interconnected system f with the help of the basic matrix of crossing circuit F^1, in two different ways.

The first method could start from the reduced matrix of the subsystem by using only two conversion matrices:

$$C_{23} = C_2 C_3, \qquad (11)$$

Let us convert the primitive matrix which is the basis for crossing circuit:

$$f' = (C_{23})_t F^{1'} C_{23}, \qquad (12)$$

f' is the matrix which defines the basic crossing circuit.

The second method to determine the matrix for an interconnected system could be carried out in one step as the matrix composition:

$$C = C_1 C_2 C_3, \qquad (13)$$

Matrix C has zero elements in rows which refer to the variables that are not related to transmission lines. Furthermore, these elements should be included into the matrix to make its size corresponding to size F^1, composted with C.

Therefore, matrix f' for basic crossing circuit could be defined as following:

$$f' = C_t F^1 C, \qquad (14)$$

IV. Introducing throughput of previously eliminated branches and final forming the matrix ***F***.

Separated branches decomposition**.** Consequently we need to use separated branches and their scalar throughput matrix which refer to the subsystem $\mathbf{J}_2$.

The final stage is to merge basic crossing circuit and separated branches:

$$F = f' + l = C_t F^1 C + l^{-1}. \qquad (15)$$

The simple increasing of separated branches of the matrix l could be done even in very complex connection types, because the quantity of rows and columns in the matrix F is equal to the quantity of the separated branches.

The matrix F conversion. The matrix F has the number of rows and columns equal to the number of open circuits. If the dimension of the matrix F is equal to the dimension of the matrix L_{ij} for subsystems, its inverted matrix could be as follows:

$$L' = F^{-1}. \qquad (16)$$

The fifth stage. The algorithm for delays matrix calculation.

For the execution of this algorithm the matrices L', F^1 and C are given for a telecommunication system that consists of $n+1$ subsystems. When loadings of the nodes in each subsystem $\mathbf{J}_1$ are equal, we could see flow balancing. And common time delay at packet transmission is equal to the time delay in the separate subsystem $\mathbf{J}_1$.

It is necessary to find vector T for time delay in nodes of the initial system. All calculations are presented in conjugated view.

The vector which we obtain at the each calculation stage has respective physical interpretation:

1. $T_1=F^{1'}H$ is the delay time for packet transmitting in separate subsystems at the given loading H in nodes.
2. $t=-C_tT_1$ is the delay time for packet transmitting in the cut **j**.
3. $h=L't$ is the loading via $\mathbf{J}_2$.
4. $H^t=Ch$ is the additional loading in subsystems $\mathbf{J}_1$, related with loading at interconnects between subsystems $\mathbf{J}_2$.
5. $T_2=F_1H^t$ is the delay time for packets into nodes, as result of the interaction with additional loading of the system $\mathbf{J}_2$.
6. $T=T_1+T_2$ is the resulting value of the delay time for transmitting packets in the merged system.

In the described algorithm the well-known procedures are combined in the new sequence to calculate complex topologies of telecommunication networks, in particular all optical networks. The diakoptics applied to the complex topological structure described by Little's equations enables to find generalized solutions for complex system considering its elementary components.

Conclusions

1. An analysis of the flow models characteristics for routing has been carried out and the ever-broadening of graph-combinatory routing models used by existent protocols has been shown.
2. The diakoptics algorithm for a complex topological structure of a telecommunication network has been proposed.
3. The generalization of the Little's equation tensor representation has been proposed to consider simultaneously the multi-flow character of the network traffic, differential packet transmitting service in network nodes, and the necessity of the QoS provision by its multiple indexes.

4. It has been shown that the diakoptics allows performance of the decomposition of the existent complex algorithms for each basic models of the flow routing.

References

1. Osterloh H. IP routing primer plus, Indianapolis, Inc, 2002.
2. Rudenko I. CISCO routers for IP networks. – Moscow: Kudis-Obraz, 2003. – 656 p.
3. Bertsekas D., Gallager R. “Data Networks,” Prentice Hall, Second Edition, 1992.
4. Vishnevskyy V.M. The theoretical basics of computer networks deploying. – Moscow: Technosfera, 2003. – 512 p.
5. Dymarskyy Y.S., Krutiakova N.P., Yanovskyy G.G. Communication networks management: principles, protocols, applied problems. – Moscow: Eco-Trends, 2003. – 384 p.
6. Lee G.M. A survey of multipath routing for traffic engineering // Proc. of LNCS 3391. Springer-Verlag, 2005. – Vol. 4. – P. 635-661.
7. Klymash M.M., Strykhalyuk B.M., Kaidan M.V. The tensor analysis of converision from traditional transport network to NGN/ Computer technologies of typography. Scientific papers digest. –Lviv, 2009. - №21. – p. 126-139.
8. Y.S. Dymarskyy, M.V. Nurmieva. Optimal resource distribution for network with heterogeneous traffic // MAISU Herald. 2002. №6. – p. 31- 35.
9. Vutukury S., Garcia-Luna-Aceves J.J. A traffic engineering approach based on minimum-delay routing // Proc. of IEEE IC3N. Las Vegas, 2000. – P. 42-47.
10. Klymash M.M., Strykhalyuk B.M., Kaidan M.V. Tensor invariance utilization for telecommunication networks modeling // Scientific papers digest „Modeling and informational technologies” IPME, NAS of Ukraine. – Kyiv, 2008. - №.49 - p.72-79.
11. Lemeshko A.V., Drobot O.A., Symonenko D.V.. Routing flowing models comparative analysis results for telecommunication networks // Scientific papers digest of Air Force Kharkiv University. №. 1(13), 2007. – p. 66-69.
12. Kleinrock L., Queueing Systems: Volume II – Computer Applications (Wiley Interscience, New York, 1976)
13. Lemeshko A.V., Drobot O.A. A multi-path QoS routing model for multi-service telecommunication network // Radio-technique: All-Ukrainian interdepartmental. Scientific and Technical digest. - 2006. – № 144. – p. 16-22.
14. Kron G. Diakoptics; the piecewise solution of large-scale systems. MacDonald, London, 1963. 166 pp.
15. Pasechnykov I.I. "Analysis and synthesis of digital communications networks on the basis of the tensor model display," in TVAYY, Tambov, 2004, 179p.
16. Kron G. Tensor analysis of networks. J.Wiley & Sons,N.Y., 1965, p.720.
17. Demydov I.V., Kryvinska N., Strauss C., Klymash M., Ivanochko I. Enterprise Distributed Service Platforms - an Approach to the Architecture and Topology // Emerging Research and Projects Applications Symposium (ERPAS 2009), in conjunction with 7th International Conference on Advances in Mobile Computing and Multimedia (MoMM2009), 14-16 December 2009, Kuala Lumpur, Malaysia, ACM ISBN: 978-1-60558-659-5 (electronic), pp. 417-421.
18. Demydov I.V., Kryvinska N., Klymash M.M. An Approach to the Flexible Information/Service Workflow Managing in Distributed Networked Architectures // The International Workshop on Design, Optimization and Management of Heterogeneous Networked Systems (DOM-HetNetS’09), in conjunction with the 38th International Conference on Parallel Processing (ICPP-2009), September 22nd -25th, 2009. - Vienna, Austria, 2009. - P. 236-242.

ПОТОКОВА МАРШРУТИЗАЦІЯ ТЕЛЕКОМУНІКАЦІЙНИХ МЕРЕЖ І ЇЇ ПРЕДСТАВЛЕННЯ МЕТОДОМ ДІАКОПТИКИ

М. Климаш, Б. Стрихалюк, М.Кайдан, І. Демидов

Класифіковано характеристики потокових моделей маршрутизації. Проведено аналіз моделей маршрутизації, які покладені в основу існуючих протоколів телекомунікаційних мереж. Проведено декомпозиційне представлення телекомунікаційних мереж і запропоновано алгоритм розрахунку методом діакоптики.

Mykhailo Klymash DSc, professor Head of Telecommunications Department, Institute of Telecommunications, Radioelectronics and Electronic Engineering, Lviv Polytechnic National University.

Bohdan Strykhalyuk PhD, assosiate-professor, Deputy-Head of Telecommunications Department, Institute of Telecommunications, Radioelectronics and Electronic Engineering, Lviv Polytechnic National University.

Mykola Kaidan PhD, assosiate - professor, Telecommunications Department, Institute of Telecommunications, Radioelectronics and Electronic Engineering, Lviv Polytechnic National University.

Ivan Demydov PhD, assistant, Telecommunications Department, Institute of Telecommunications, Radioelectronics and Electronic Engineering, Lviv Polytechnic National University.