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THE PRECISE SOLUTION OF THE ALLOWING FOR THE HIGH ORDER REFRACTIVE EFFECTS ON DUAL-FREQUENCY GNSS MEASUREMENTS

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Among the actual research directions in the field of Global Navigation Satellite Systems (GNSS) the studies of the accuracy of measurements performed with the help of such systems take an important place. The improvement of GNSS measurements accuracy is one of the progress conditions in such spheres of activity as geodynamic researches [1], realization and maintenance of International Terrestrial Reference Frame, atmospheric researches, metrology and fundamental researches (for example, checking the time and frequency measurement standards, studying gravitational and relativistic effects [23]).

One of the major factors, limiting the accuracy of GNSS measurements, is the influence of the Earth's atmosphere on the characteristics of GNSS signals distribution. The main source of measurement errors in this case appears to be additional delay of a signal in atmosphere and refractive bending of the signal trajectory. The Earth's atmosphere influence can be compensated by means of either injecting of the appropriate corrections while processing observed data, or in an instrument way – by using the measuring information obtained immediately from measurements [4, 5]. The influence of the troposphere is taken into account, as a rule, by introduction the tropospheric corrections to the set of unknowns determined from results of the GNSS measurements [4]. To reduce the influence of ionosphere instrumentally the dual-frequency method is traditionally used. This method is based on the fact the ionosphere is a dispersive medium, i.e. the speed of a signal propagating through the ionosphere depends on its frequency [5]. To obtain the measurement equation this method uses the following system of the simplified equations

$$\begin{cases} P_1 = L + \dfrac{a}{f_1^2} \int\limits_{\sigma} N_e d\sigma \\ P_2 = L + \dfrac{a}{f_2^2} \int\limits_{\sigma} N_e d\sigma \end{cases},$$

where P_1, P_2 are the measured pseudo-ranges [5], and the distance L between the satellite and a receiver and the integral of the electron concentration N_e along a straight line between them, (fig. 1a) are determinable.

The dual-frequency method is approximate, it allows to eliminate the contribution to the observation result of the only part of a signal ionospheric delay determined by the main term of series expansion of an ionosphere refractive index n_{ion}, inversely proportional to the square of frequency f (see fig. 1a) [5, 6].

The higher order terms of series expansion are not taken into account. Also at the derivation of the measurement equation of the classical dual frequency method the refractive effect of space separating of the ray paths with different carrier frequencies, and bounded with this effect the difference in signal tropospheric delays are not taken into account (fig. 1б) [6, 7]. According to [6, 8], we shall further name the listed above effects as higher order effects.

The mentioned above simplifications lead to the principal limitations of accuracy in the classical dual-frequency method. The performed estimations point to the essential role of the higher order effects for the accuracy of GNSS measurements (contribution of the mentioned effects to the pseudo-range for elevation angles of about 10 degrees can give from 1 up to 7 cm depending on measurements conditions [6, 8-10]). Now the project [11] has already started, with the goal to reprocess the data mined from the global GPS network over the time span from 1994 up to present time allowing for the different higher order effects. Strict requirements to the reprocessing accuracy, such that the site coordinate error should be no more than 1 mm, make the registration of such effects a mandatory condition. At the same time, in the most of works devoted to the mentioned problem, the approximate methods are developed: thus refractive effects of spatial separating of the ray paths with different carrier frequencies and their refractive lengthening due to bending are usually not considered, neither the interference of tropospheric and ionospheric effects is taken into account. [9].

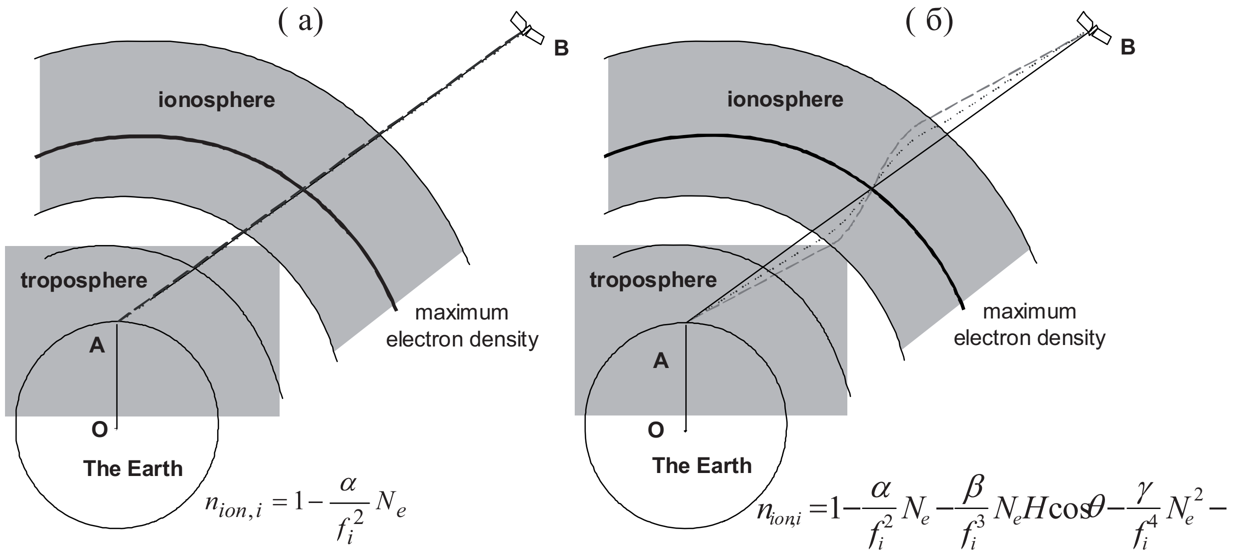

Fig. 1. The simplified scheme of propagation of the signals with different frequencies in the atmosphere and series expansion of the refractive index n_{ion} up to the first order terms used for derivation of the measurement equation of the classical dual-frequency method (a), and secondly the scheme of signal propagation and the form of the refractive index for derivation of the measurement equation allowing for the higher order ionospheric effects (б)

In the present paper the basis of precise expressions to find out the corrections (which take into account the contribution of the higher order ionospheric effects at dual-frequency GNSS measurements, including refractive effects of lengthening due to bending and spatial separating of the ray paths with different carrier frequencies while propagating through ionosphere and troposphere) is set.

The basis of the precise correction equations

To derive the measurement equation for the dual-frequency method with the corrections taking into account higher order refractive ionospheric effects, the following input equations were used. First of all, these are expressions for pseudo-ranges P_i on frequencies f_i ($i = 1, 2$ for the dual-frequency method), which we present as [5]

$$P_i = \rho_i + \Delta P_{trop,i} + \Delta P_{ion,i}, \quad i = 1, 2. \qquad (1)$$

where

$$\rho_i = \int_{\sigma(f_i)} d\sigma$$

is the length of the ray path for frequency f_i;

$$\Delta P_{trop,i} = \int_{\sigma(f_i)} (n_{trop} - 1) d\sigma$$

is the delay in troposphere on frequency f_i;

$$\Delta P_{ion,i} = \int_{\sigma(f_i)} (n_{ion,i} - 1) d\sigma$$

is the delay in ionosphere on frequency f_i.

Integration for each of frequencies f_1 and f_2 is carried out along the trajectories $\sigma(f_1)$ and $\sigma(f_2)$ accordingly (the equations to compute the trajectories, the explicit view of which is not used in the present paper, are done in many publications, for example, in [12]). The values n_{trop}, $n_{ion,i}$ are the refractivity indexes accordingly of troposphere (see, for example, [13]) and ionosphere for frequency f_i in a current point of the signal trajectory. The formula for $n_{ion,i}$ (see fig. 1б) can be represented as a series expansion [6]:

$$n_{ion,i} = 1 - \frac{\alpha}{f_i^2} N_e - \frac{\beta}{f_i^3} N_e H \cos\theta - \frac{\gamma}{f_i^4} N_e^2 - R_i,$$

where N_e is the electron density; H is the amplitude of the geomagnetic field, θ is the angle between the geomagnetic field vector H and the propagation direction of the ray path, α, β, γ are the constant coefficients, which include the fundamental constants [6], R_i denotes the remainder term of the series expansion.

Then

$$\Delta P_{ion,i} = \rho_i - \frac{\alpha}{f_i^2} \int\limits_{\sigma(f_i)} N_e d\sigma - \frac{\beta}{f_i^3} \int\limits_{\sigma(f_i)} N_e H \cos\theta d\sigma - \frac{\gamma}{f_i^4} \int\limits_{\sigma(f_i)} N_e^2 d\sigma - \int\limits_{\sigma(f_i)} R_i d\sigma \quad . \qquad (2)$$

In the equation (1) we have added tropospheric delay, which is not considered at derivation of the classical dual-frequency method, to take into account the difference of delays in troposphere, bounded with the refractive spatial separating of the ray paths due to propagation through ionosphere.

Let's enter a lengthening of trajectories due to the refraction in an inhomogeneous medium:

$$\delta\rho_i = \rho_i - L, \qquad i = 1, 2, \qquad (3)$$

Using the equations of sort (1) on two frequencies and expressions (2)–(3), we obtain the following set of equations

$$\begin{cases} P_1 = L + \delta\rho_1 + \Delta P_{trop,1} + \dfrac{a}{f_1^2} \int\limits_{\sigma(f_1)} N_e d\sigma \\ \quad + \dfrac{\beta}{f_1^3} \int\limits_{\sigma(f_1)} N_e H \cos\theta d\sigma + \dfrac{\gamma}{f_1^4} \int\limits_{\sigma(f_1)} N_e{}^2 d\sigma + \int\limits_{\sigma(f_1)} R_1 d\sigma \\ P_2 = L + \delta\rho_2 + \Delta P_{trop,2} + \dfrac{a}{f_2^2} \int\limits_{\sigma(f_2)} N_e d\sigma \\ \quad + \dfrac{\beta}{f_2^2} \int\limits_{\sigma(f_2)} N_e H \cos\theta d\sigma + \dfrac{\gamma}{f_2^4} \int\limits_{\sigma(f_2)} N_e{}^2 d\sigma + \int\limits_{\sigma(f_2)} R_2 d\sigma \end{cases} .$$

Having conversed this system the similar way as derivation of the classical dual-frequency method's equation, we obtain the measurement equation of the dual-frequency method allowing for the higher order tropospheric and ionospheric effects:

$$L = P_1 - \frac{f_2^2}{f_1^2 - f_2^2}(P_2 - P_1) - \Delta P_{trop,1} + \delta_\Sigma . \qquad (4)$$

The first two addends in the right part of (4) correspond to the measurement equation of the classical dual-frequency method implementing the so-called ionosphere-free combination [5]; $\Delta P_{trop,1}$ is the tropospheric delay; δ_Σ is the aggregate correction taking into account higher order effects.

The strict formula for the correction δ_Σ can be presented in a different view, for example, as:

$$\begin{aligned} \delta_\Sigma = -\delta\rho_1 + \frac{f_2^2}{f_1^2 - f_2^2}[(\delta\rho_2 - \delta\rho_1) + \\ + (\Delta P_{trop,2} - \Delta P_{trop,1}) + \\ + (\Delta P_{ion,2} - \frac{f_1^2}{f_2^2} \Delta P_{ion,1})] \quad . \end{aligned} \qquad (5)$$

The analysis of the correction taking into account higher order effects.

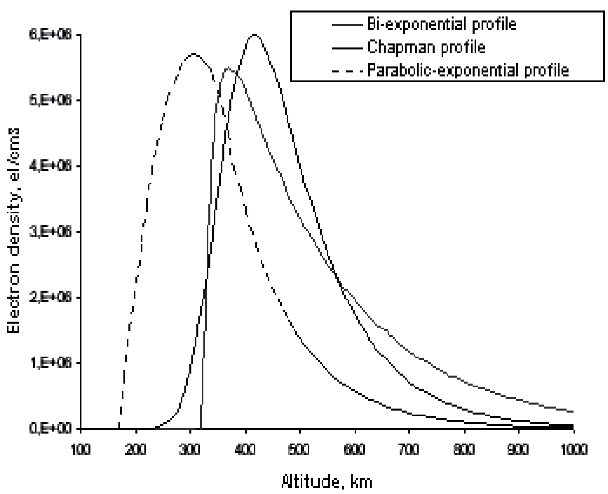

Fig. 2. Profiles of electron density distribution with different values and heights of maximum electron concentration and similar TEC

For practical usage of the formula (5) it is necessary to have an algorithm to find out entry values. The estimations of the correction value taking into account higher order effects (and its components, included in the formula (5)), are determined using the modeling of electron density distribution along the ray path of a signal. The results of calculations performed for ionospheric parameters, used in [6] (at an elevation angle of 15 degrees for TEC=138 TECU), are shown in the Table 1 and Table 2, where the contribution of lengthening $\delta\rho_1$ on f_1 is denoted as $\delta 1$, the contribution of the difference of delays in troposphere $\frac{f_2^2}{f_1^2 - f_2^2}\left[\Delta P_{trop,2} - \Delta P_{trop,1}\right]$ is denoted as $\delta 2$, the remaining addends of the formula (5) are grouped together under the name of $\delta 3$.

At an elevation angle of 10°, which is used at GNSS measurements enough frequently, the value of the correction will be even bigger. In particular, the contribution of the difference of tropospheric delays caused by refractive spatial separating of the ray paths at an elevation angle of 10° for various models of electron density distribution (see Fig. 2) with the similar total electron contents (TEC) is represented in the table 3.

Table 1

Values of the correction components for the Chapman ionospheric profile at quasilongitudinal propagation of a signal in the geomagnetic field

Measurements	δ_{Σ}, cm	$\delta 1$, cm	$\delta 2$, cm	$\delta 3$, cm
Code	8,27	1,75	0,13	9,79
Phase	-4,65	1,65	0,13	-3,13

Table 2

Values of the correction components for the Chapman ionospheric profile at quasitransversal propagation of a signal in the geomagnetic field

Measurements	δ_{Σ}, cm	$\delta 1$, cm	$\delta 2$, cm	$\delta 3$, cm
Code	3,10	1,65	0,13	4,62
Phase	-2,07	1,65	0,13	-0,55

Table 3

The contribution of the difference of tropospheric delays at the elevation angle of 10°

Model and TEC	$\delta 2$, mm
Bi-exponential [14], 142 TECU	3,39
Chapman profile [6], 137 TECU	3,56
Parabolic-exponential [14], 135 TECU	5,16

The bigger the maximum of electron concentration and the more “acute” form of the profile (with similar TEC), the bigger the contribution of the difference of tropospheric delays caused by refractive spatial separating of the ray paths.

Discussion on the obtained results

Against to the approximate expressions of the known works, the obtained in this paper formula (5) expresses the corrections of the dual-frequency method through delays of signals and lengthening of their trajectories on each frequency with the help of strict formulas which take into account all higher orders effects correctly. It is obvious, that the contribution of the difference of tropospheric delays, which haven’t been taken into account till now in works on modeling of higher orders effects, can be enough essential. Therefore it’s important to use the measurement equation (4) together with the formula (5) as the initial expressions, if the purpose is to increase the accuracy of measurements performed by the dual-frequency method, as these expressions give the possibility to take into account the contribution to uncertainty of all influencing factors precisely.

Conclusions

The strict expressions to calculate the corrections to results of the dual-frequency GNSS measurements which take into account refractive higher order effects, caused by contribution of additional terms of series expansion of an ionosphere refractive index on return degrees of frequency, and also effects of the refractive lengthening and refractive space separating of the ray paths with different carrier frequencies at their propagation through ionosphere and troposphere (and difference, caused by the given effects, in tropospheric delays of these signals).

The obtained expressions can be used for increasing the accuracy of measurements performed by the dual-frequency method, in particular, at reprocessing of GPS measurements’ results in IGS network [11].

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Точне розв’язання задачі урахування рефракційних ефектів вищих порядків під час двочастотних ГНСС-вимірювань

А. Олійник, О. Прокопов, І. Тревого

Запропоновано точне розв’язання задачі урахування рефракційних іоносферних ефектів вищих порядків, зумовлених внеском членів розкладання коефіцієнта заломлення іоносфери другого і третього порядків, а також ефектів рефракційного подовження і рефракційного просторового рознесення траєкторій сигналів з різними несучими частотами при їхньому поширенні в іоносфері і тропосфері під час двочастотних ГНСС-вимірювань.

Точное решение задачи учета рефракционных эффектов высших порядков при двухчастотных ГНСС-измерениях

А. Олейник, А. Прокопов, И. Тревого

Предложено точное решение задачи учета рефракционных ионосферных эффектов высших порядков, обусловленных вкладом членов разложения коэффициента преломления ионосферы второго и третьего порядков, а также эффектов рефракционного удлинения и рефракционного пространственного разнесения траекторий сигналов с различными несущими частотами при их распространении в ионосфере и тропосфере при проведении двухчастотных ГНСС-измерений.

The precise solution of the allowing for the high order refractive effects on dual-frequency GNSS measurements

A. Oliinyk, A. Prokopov, I. Trevogo

The precise solution of the allowing for the high order refractive ionospheric effects caused by not taking into account the second and third order terms of refractive index series expansion, as well as the refractive effects of lengthening due to bending and spatial separating of the ray paths with different carrier frequencies while propagating through ionosphere and troposphere on dual-frequency GNSS measurements.